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Indiana's Academic Standards—Mathematics

CORE STANDARDS

KINDERGARTEN

CORE STANDARD 1

Number Sense and Computation

Counting to 20

Count objects and use objects, pictures and numerals to represent whole numbers up to 20. Find the number that is one more than or one less than any whole number up to 20. Recognize numbers from 10 to 20 and represent them as groups of tens and ones using objects, diagrams and numerals.

[Standard Indicators: K.1.1, K.1.2, K.1.3, K1.4]

CORE STANDARD 2

Geometry and Measurement

Comparing and Classifying Objects

Identify, describe, sort, compare and classify objects by shape, size, number of vertices and other attributes.

[Standard Indicators: K.3.1]

Comparing Measures

Make direct comparisons of the length and weight of objects and recognize which object is shorter, longer, taller, lighter or heavier.

[Standard Indicator: K.3.3]

GRADE 1

CORE STANDARD 1

Number Sense and Computation

Whole Numbers

Count, read, write and compare whole numbers up to 100. Represent numbers up to 100 as groups of tens and ones.

[Standard Indicators: 1.1.1, 1.1.4,]

Addition and Subtraction

Model addition and subtraction using objects. Demonstrate fluency with addition facts and the corresponding subtraction facts for totals up to 20. Solve problems involving addition and subtraction.

[Standard Indicators: 1.1.5, 1.1.6]

CORE STANDARD 2

Geometry and Measurement

Geometric Shapes

Identify, describe, compare, sort and draw triangles, rectangles, squares and circles.
[Standard Indicators: 1.3.1, 1.4.2, 1.4.7]

Linear Measurement

Estimate and measure the length of an object to the nearest inch and centimeter.
[Standard Indicator: 1.3.2]

GRADE 2

CORE STANDARD 1

Number Sense and Computation

Place Value

Understand and use the relationship among whole numbers, including place value, to identify and compare numbers up to 1000.
[Standard Indicators: 2.1.1, 2.1.2, 2.1.4,]

Addition and Subtraction

Add and subtract whole numbers less than 1000 using efficient methods. Understand and show the inverse relationship between addition and subtraction.
[Standard Indicators: 2.1.6]

CORE STANDARD 2

Geometry and Measurement

Common Shapes and Objects

Recognize, identify, and describe attributes of common shapes and solids (e.g., the same size and type of shape; number of sides, edges, vertices; location in space).
[Standard Indicators: 2.3.1]

Linear Measurement

Measure lengths in standard units (e.g., inches, feet and yards) and metric units (e.g., centimeters and meters) and select appropriate units to estimate and measure lengths. Understand and use units of linear measurement and relationships within a particular system to solve problems.
[Standard Indicators: 2.3.3]

GRADE 3

CORE STANDARD 1

Number Sense and Computation

Place Value

Understand and use the relationship among whole numbers, including place value, to identify and compare numbers up to 10,000.

[Standard Indicators: 3.1.1, 3.1.4]

Interpreting and Modeling Fractions

Interpret and model fractions as parts of a whole, parts of a group, and points and distances on a number line. Solve problems that involve comparing and ordering fractions.

[Standard Indicators: 3.1.2, 3.1.3]

Addition and Subtraction

Understand and use standard algorithms for addition and subtraction.

[Standard Indicators: 3.1.5]

Multiplication and Division

Understand the meaning of multiplication and division of whole numbers and show the relationship between them.

[Standard Indicators: 3.1.6, 3.1.7, 3.2.1, 3.2.2, 3.2.3]

CORE STANDARD 2

Geometry and Measurement

Points and Lines

Identify, describe and draw points, lines and line segments.

[Standard Indicators: 3.3.2]

Length, Weight and Unit Conversions

Choose and use appropriate units and tools to estimate and measure length and weight. Use the relationship between the units to express answers in different units.

[Standard Indicators: 3.3.5]

GRADE 4

CORE STANDARD 1

Number Sense and Computation

Place Value

Understand and use the relationship among whole numbers, including place value, to identify and compare numbers. Interpret and model decimals as parts of a whole, parts of a group, and points and distances on a number line.

[Standard Indicators: 4.1.1, 4.1.3, 4.1.4]

Multiplication and Division Facts

Demonstrate fluency with multiplication facts for numbers up to 10 and related division facts.

[Standard Indicator: 4.1.5]

Multiplying Whole Numbers

Multiply numbers up to 100 by single-digit numbers and two-digit numbers.
[Standard Indicators: 4.1.6, 4.2.2, 4.2.3]

Addition and Subtraction of Fractions

Model addition and subtraction of simple fractions.
[Standard Indicator: 4.1.7, 4.1.2]

CORE STANDARD 2

Geometry and Measurement

Angles and Lines

Identify, describe and draw parallel and perpendicular lines and right, acute, obtuse and straight angles.
[Standard Indicators: 4.3.1, 4.3.2]

Rectangles

Find and use the perimeter and area of rectangles, including squares.
[Standard Indicators: 4.3.5]

GRADE 5

CORE STANDARD 1

Number Sense and Computation

Multiplication and Division of Whole Numbers

Understand and use standard algorithms for multiplication and division of whole numbers.
[Standard Indicator: 5.1.5]

Comparing Fractions and Decimals

Compare fractions, and decimals.
[Standard Indicators: 5.1.1, 5.1.2]

Addition and Subtraction of Fractions and Decimals

Understand and perform addition and subtraction with fractions, including fractions with different denominators and mixed numbers. Add and subtract decimals, including money in decimal notation.
[Standard Indicators: 5.1.6]

CORE STANDARD 2

Geometry and Measurement

Polygons

Measure angles and describe angles in degrees and identify, classify and draw polygons and triangles.
[Standard Indicators: 5.3.1, 5.3.2]

Shapes and Solids

Find and use the perimeter and area of triangles, parallelograms and trapezoids, and the surface area and volume of rectangular prisms.

[Standard Indicators: 5.3.5, 5.3.6]

CORE STANDARD 3

Algebra and Functions

Variables

Evaluate simple algebraic expressions.

[Standard Indicators: 5.2.1]

Coordinate Grids

Use two-dimensional coordinate grids to represent points in the first quadrant that fit linear equations and draw the line determined by the points.

[Standard Indicators: 5.2.2]

GRADE 6

CORE STANDARD 1

Number Sense and Computation

Positive and Negative Numbers

Understand and apply the concept of positive and negative numbers. Add, subtract, multiply and divide positive and negative integers. Represent negative numbers, and computation with negative numbers, on a number line.

[Standard Indicators: 6.1.1, 6.1.5]

Percent Representations

Use percents to represent parts of a whole. Represent numbers as fractions, decimals and percents.

[Standard Indicators: 6.1.3, 6.1.4]

Multiplication and Division of Fractions and Decimals

Understand and perform multiplication and division with positive decimals and fractions.

[Standard Indicators: 6.1.6]

Ratio and Rate

Solve simple ratio and rate problems using multiplication and division.

[Standard Indicators: 6.1.7, 6.1.9]

CORE STANDARD 2

Geometry and Measurement

Angles and Polygons

Use properties of complementary, supplementary and vertical angles, and properties of triangles and quadrilaterals, to find missing angles.

[Standard Indicators: 6.3.1, 6.3.2]

Shapes and Solids

Find and use the circumference and area of circles and the surface area of right prisms and cylinders.

[Standard Indicators: 6.3.3, 6.3.5]

CORE STANDARD 3

Algebra and Functions

Linear Equations

Write and solve one-step equations and inequalities in one variable.

[Standard Indicators: 6.2.1, 6.2.3]

Linear Functions

Use equations and graphs of linear functions to represent a given situation.

[Standard Indicators: 6.2.4, 6.2.5]

GRADE 7

CORE STANDARD 1

Number Sense and Computation

Exponents

Use whole number exponents for repeated multiplication. Use scientific notation for large numbers.

[Standard Indicators: 7.1.1, 7.1.2, 7.1.3]

Proportions and Percentages

Use proportions and percentages to solve application problems involving the increase of a quantity and the decrease of a quantity. Solve problems involving percent, ratio, rate and similar triangles.

[Standard Indicators: 7.1.8, 7.1.9]

Multiplication and Division with Fractions and Decimals

Understand and perform multiplication and division with negative decimals and fractions.

[Standard Indicator: 7.1.7]

CORE STANDARD 2

Geometry and Measurement

Transformations

Identify and use the following transformations: translations, rotations and reflections.
[Standard Indicator: 7.3.1]

CORE STANDARD 3 **Algebra and Functions**

Expressions

Evaluate numerical expressions and simplify algebraic expressions involving rational and irrational numbers.

[Standard Indicators: 7.2.1, 7.2.3]

Linear Equations

Write and solve two-step equations and inequalities in one variable.

[Standard Indicators: 7.2.1, 7.2.2]

Graphs of Lines

Find the slope of a line from its graph and relate the slope of a line to similar triangles. Draw the graph of a line given its slope and one point on the line or two points on the line. Graph proportional relationships and identify the unit rate as the slope of the related line.

[Standard Indicators: 7.2.5, 7.2.6, 7.2.7]

CORE STANDARD 4 **Data Analysis and Probability**

Making Estimates and Data Displays

Use proportions to make estimates about a population based on a sample. Create, analyze and interpret data sets in multiple ways using bar graphs, frequency tables, line plots, histograms and circle graphs.

[Standard Indicators: 7.4.3, 7.4.1, 7.4.3, 7.4.4]

Theoretical Probability

Understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Use theoretical probability and proportions to make predictions.

[Standard Indicators: 7.4.5]

GRADE 8

CORE STANDARD 1 **Number Sense and Computation**

Integer Exponents

Use the laws of integer exponents and evaluate expressions with negative integer exponents. Use scientific notation for small numbers.

[Standard Indicators: 8.1.3, 8.1.1, 8.1.2]

Square Roots

Use irrational numbers. Calculate square roots. Use the inverse relationship between squares and square roots.

[Standard Indicators: 8.1.4, 8.1.5]

CORE STANDARD 2

Geometry and Measurement

Constructions and Properties of Shapes

Perform basic compass and straightedge constructions, such as constructions of angle and segment bisectors, copies of segments and angles, and perpendicular segments. Justify the constructions. Identify properties of geometric shapes.

[Standard Indicators: 8.3.1, 8.3.2]

Pythagorean Theorem

Use the Pythagorean Theorem and its converse to calculate lengths of line segments.

[Standard Indicator: 8.3.4]

Rates

Solve simple problems involving rates and derived measurements such as speed and density. Express these measurements in a given unit in terms of other units within the same measurement system.

[Standard Indicators: 8.3.4, 8.3.10]

Solids

Find and use the surface areas and volumes of cones, spheres and pyramids. Use scale factors to find areas and volumes of similar figures.

[Standard Indicators: 8.3.5, 8.3.6]

CORE STANDARD 3

Algebra and Functions

Solving Equations and Inequalities

Write and solve multi-step equations and inequalities in one variable.

[Standard Indicators: 8.2.1]

Linear Functions

Use linear functions and linear equations to represent, analyze and solve problems. Translate among tables, equations, verbal expressions and graphs.

[Standard Indicators: 8.2.4, 8.2.5, 8.2.6, 8.2.8]

CORE STANDARD 4

Data Analysis and Probability

Analyzing Data

Use mean, median, mode, upper and lower quartiles, and range of data to compare data sets. Organize and display data to analyze central tendencies of data. Investigate effects of change in data values on the measures of the central tendency of the set of data. Display data in scatter plots, informally finding lines of best fit.

[Standard Indicators: 8.4.3, 8.4.5]

Evaluating Claims, Selecting Samples and Analyzing Bias

Identify claims based on statistical data and, in simple cases, evaluate the reasonableness of the claims. Identify different methods of selecting samples, analyzing the strengths and weaknesses of each method and the possible bias in a sample or display.

[Standard Indicators: 8.4.1, 8.4.2]

Simple Experiments

Compute probabilities of events from simple experiments with equally probable outcomes.

[Standard Indicator: 8.4.7]

ALGEBRA I

CORE STANDARD 1

Rational Exponents

Understand and use the laws of exponents for variables with exponents. Multiply, divide, and find powers of variables with exponents.

[Standard Indicators: A1.4.1]

CORE STANDARD 2

Relations and Functions

Determine whether a relation is a function or not a function. Identify the domain and range of a given relation. Translate among tables, graphs, words and equations.

[Standard Indicators: A1.1.1, A1.1.2, A1.2.1]

CORE STANDARD 3

Linear Equations and Inequalities

Graphing and Writing Linear Equations

Graph linear functions and determine their slopes and x - and y -intercepts from their graphs and equations. Write a linear function in slope-intercept form. Determine the equation of a line given sufficient information.

[Standard Indicators: A1.2.2, A1.2.3, A1.2.4]

Pairs of Linear Equations in Two Variables

Solve pairs of linear equations in two variables by graphing, substitution or elimination. Solve problems that can be modeled using pairs of linear equations in two variables.

[Standard Indicators: A1.3.1, A1.3.3]

Pairs of Linear Inequalities in Two Variables

Graph the solution for pairs of linear inequalities in two variables.
[Standard Indicator: A1.3.2]

CORE STANDARD 4

Polynomials

Multiply polynomials, factor polynomials, and divide a polynomial by a monomial.
[Standard Indicators: A1.4.2, A1.4.3]

CORE STANDARD 5

Quadratic Equations and Functions

Solve quadratic equations by graphing, factoring, and using the quadratic formula. Graph quadratic functions and understand the relationship between its zeros and the x -intercepts of its graph. Solve problems that can be modeled using quadratic equations.
[Standard Indicators: A1.5.1, A1.5.2, A1.5.3, A1.5.4]

GEOMETRY

CORE STANDARD 1

Angles and Lines

Understand the relationship between special angles created by parallel lines and transversals.
[Standard Indicator: G.1.3]

CORE STANDARD 2

Congruence and Similarity

Develop simple geometric proofs involving congruent and similar polygons. Solve problems involving congruent and similar polygons and solids.
[Standard Indicators: G.2.3, G.2.7, G.2.12, G.4.2]

CORE STANDARD 3

Triangles

Prove the Pythagorean Theorem and its converse and use them to solve problems. Develop simple geometric proofs involving triangles. Define trigonometric functions in terms of angles of right triangles and use them to solve problems.
[Standard Indicators: G.2.16, G.2.17, G.2.14, G.2.19, G.2.21]

CORE STANDARD 4

Polygons

Find the sum of the measures of the interior and exterior angles of convex polygons. Deduce formulas relating lengths and sides, perimeters, and areas of regular polygons, and understand how limiting cases of such formulas lead to expressions for the circumference and the area of a circle.
[Standard Indicators: G.2.1, G.2.5]

CORE STANDARD 5

Circles

Define, deduce and use formulas for, and prove theorems for:

- radius, diameter, arc, chord, secant and tangent;
- measures of arcs and related angles (central, inscribed, and intersections of secants and tangents); and
- circumference, arc length and areas of circles and sectors.

Determine how the graph of a circle changes if a, b, and r are changed in the equation $(x - a)^2 + (y - b)^2 = r^2$.

[Standard Indicators: G.3.2, G.3.3, G.3.4, G.3.5, G.3.6]

CORE STANDARD 6

Solids

Find and use measures of sides, volumes of solids, and surface areas of solids. Relate these measures to each other using formulas.

[Standard Indicator: G.4.3]

CORE STANDARD 7

Coordinate Geometry

Find slopes, lengths and midpoints of line segments using coordinate geometry. Use these measures to show whether shapes are similar or congruent, and whether line segments are parallel or perpendicular. Find the equation of a circle in the coordinate plane.

[Standard Indicators: G.1.1, G.1.4, G.1.6, G.3.5]

CORE STANDARD 8

Transformations

Predict and describe the results of translations, reflections and rotations. Describe a motion or series of motions that will show that two shapes are congruent.

[Standard Indicator: G.2.4]

CORE STANDARD 9

Geometric Proof and Reasoning

Understand the difference between supporting evidence, counterexamples and actual proofs. Be able to develop simple geometric proofs involving the following topics and provide reasons for each statement:

- parallel lines and transversals;
- congruent and similar polygons, particularly triangles;
- circles; and
- geometric objects in the coordinate plane.

[Standard Indicators: G.2.7, G.2.14, G.3.6, G.5.3, G.5.4]

ALGEBRA II

CORE STANDARD 1

Linear and Absolute Value Equations and Inequalities

Solve systems of linear equations and inequalities in three variables by substitution and elimination. Solve problems that can be modeled using systems of linear equations. Solve equations and inequalities involving absolute value.

[Standard Indicators: A2.2.1, A2.2.2, A2.2.4]

CORE STANDARD 2

Complex Numbers

Add, subtract, multiply and divide complex numbers.

[Standard Indicator: A2.3.1]

CORE STANDARD 3

Quadratic Equations and Functions

Solving Quadratic Equations

Solve quadratic equations in the complex number system. Solve problems that can be modeled using quadratic equations and functions.

[Standard Indicators: A2.3.2, A2.3.5]

Graphing Quadratic Functions

Graph quadratic functions. Determine how the graph of a parabola changes if a , b , and c are changed in the equation $y = a(x - b)^2 + c$. Find an equation for a parabola given sufficient information.

[Standard Indicator: A2.3.3, A2.3.4]

CORE STANDARD 4

Polynomial Equations and Functions

Solving Polynomial Equations

Solve polynomial equations by factoring. Solve problems that can be modeled using polynomial equations.

[Standard Indicators: A2.4.4, A2.4.6]

Writing Polynomial Equations

Perform arithmetic operations, including long division, on polynomials. Find a polynomial given its roots and use the relationship between solutions of an equation, zeros of a function, x -intercepts of a graph and factors of a polynomial expression to solve problems.

[Standard Indicators: A2.4.3, A2.4.7]

CORE STANDARD 5

Rational Functions

Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions, including expressions with negative exponents in the denominator. Solve problems that can be modeled using equations involving rational functions.

[Standard Indicators: A2.5.2]

CORE STANDARD 6

Exponential and Logarithmic Equations

Use laws of exponents to derive laws of logarithms. Use laws of logarithms to solve problems. Solve exponential and logarithmic equations. Solve problems that can be modeled using equations involving exponents and logarithms.

[Standard Indicators: A2.6.2, A2.6.3, A2.6.4]

CORE STANDARD 7

Sequences and Series

Find specific terms of arithmetic and geometric sequences. Find partial sums of arithmetic and geometric series. Solve problems that can be modeled using arithmetic and geometric series.

[Standard Indicators: A2.7.1, A2.7.3, A2.7.4]

CORE STANDARD 8

Combinatorics and Probability

Use permutations, combinations and other counting methods to determine the number of ways that events can occur. Calculate the probability of compound events, and analyze probabilities to interpret odds and risks of events.

[Standard Indicators: A2.8.2, A2.8.4]

PRE-CALCULUS

CORE STANDARD 1

Graphing Functions

Use paper and pencil methods and graphing technology to graph polynomial, absolute value, rational, algebraic, exponential, logarithmic, trigonometric and inverse trigonometric functions. Identify domain, range, intercepts, zeros, asymptotes and points of discontinuity of functions. Use graphs to solve problems.

[Standard Indicators: PC.1.1, PC.1.2, PC.1.3, PC.3.1, PC.3.2, PC.3.3, PC.4.8, PC.4.9, PC.4.10]

CORE STANDARD 2

Conic Sections

Derive equations for conic sections. Graph conic sections by hand by completing the square and find foci, centers, asymptotes, eccentricity, axes and vertices (as appropriate).

[Standard Indicator: PC.2.1, PC.2.2]

CORE STANDARD 3

Logarithmic and Exponential Functions

Define and find inverse functions. Verify whether two given functions are inverses of each other. Solve problems involving logarithmic and exponential functions using the laws of logarithms and understand why those properties are true.

[Standard Indicators: PC.1.7, PC.3.2]

CORE STANDARD 4

Unit Circle

Define sine and cosine using the unit circle, converting between degree and radian

measures. Use the values of the sine, cosine and tangent functions at 0 , $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$ and $\frac{\pi}{2}$ radians and their multiples.

[Standard Indicators: PC.4.4, PC.4.5, PC.4.6]

CORE STANDARD 5

Trigonometric Functions

Define and analyze trigonometric functions, including inverse functions. Solve problems involving trigonometric functions and prove trigonometric identities.

[Standard Indicators: PC.4.8, PC.4.9, PC.4.10, PC.4.11]

CORE STANDARD 6

Polar Coordinates and Complex Numbers

Define and use polar coordinates and complex numbers. Graph equations in the polar coordinate plane. Use their relation to trigonometric functions to solve problems.

[Standard Indicators: PC.5.1, PC.5.2, PC.5.3, PC.5.4]

CORE STANDARD 7

Sequences and Series

Define arithmetic and geometric sequences and series. Prove and use the sum formulas for arithmetic series and for finite and infinite geometric series. Describe the concept of the limit of a sequence and a limit of a function. Decide whether simple sequences converge or diverge, and recognize an infinite series as the limit of a sequence of partial sums. Use series to solve problems. Derive the binomial theorem by combinatorics.

[Standard Indicators: PC.7.1, PC.7.2, PC.7.3]

PROCESS STANDARDS

Indiana’s Standards for Mathematics describe the key content for students in each grade level and course. Students must develop conceptual understanding of this content. The American Diploma Project noted that, “beyond acquiring procedural mathematical skills with their clear methods and boundaries, students need to master the more subjective skills of reading, interpreting, representing and ‘mathematicizing’ a problem” (p. 55).

The National Council of Teachers of Mathematics has described five Process Standards that “highlight ways of acquiring and using content knowledge” (p. 29). The following Process Standards must be addressed throughout the learning and teaching of Indiana’s Academic Standards for Mathematics in all grade levels in mathematics.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Round numbers to a specified place value.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurements in problem situations.
- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication, and problem solving.
- The challenge is to ensure that technology supports-but is not a substitute for- the development of skills with basic operations, quantitative reasoning, and problem-solving skills.
 - Elementary students should learn how to perform thoroughly the basic arithmetic operations independent of the use of a calculator.
 - Graphing calculators should be used to enhance middle school and high school students' understanding and skills.
 - The focus must be on learning mathematics, using technology as a tool rather than as an end in itself.

References

American Diploma Project (2004). *Ready or not: Creating a high school diploma that counts*. Washington, DC: Achieve, Inc.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston VA: author.

KINDERGARTEN

Standard 1 **Number Sense and Computation**

K.1.1 Count objects in a set and use objects, pictures and numerals to represent whole numbers to 20.

Example: Children match the oral counting numbers with each item in a set and give the last number counted as the quantity of the set. Students recognize five objects, five pictures, the word five and the numeral 5 as equivalent.

K.1.2 Find the number that is one more than or one less than any whole number up to 20.

Example: Play “One Less” Dominoes by placing a domino that represents one less than the last piece played and saying, “Five is one less than six,” as they place their piece. Play “One More” Dominoes also.

K.1.3 Use correctly the words one/many, none/some/all, more/less, most/least, and equal to/more than/less than.

Example: Take some of the blocks out of the box but not all of the blocks.

K.1.4 Show equivalent forms of whole numbers from 10 to 20 as groups of tens and ones using objects, diagrams and numerals.

Example: Using a two-column mat, put ten counters on the left side and five counters on the right side. Identify the number of counters by saying ten and five is fifteen. Vary the number of ones and repeat.

K.1.5 Model addition by joining sets of objects (for any two sets with fewer than 10 objects when joined) and model subtraction by removing objects from sets for numbers less than 10.

Examples:

- Join a group of three cubes to a group of four cubes to make a group of seven cubes while saying $7 = 4 + 3$.
- From a pile of eight crayons remove six crayons and tell how many crayons are left, $8 - 6 = 2$.

K.1.6 Record and organize information and answer questions about data using objects and pictures in context.

Example: Ask everyone in your class which color is their favorite. Color one box on grid paper for each child’s response.

Standard 2 **Algebra and Functions**

K.2.1 Verbally describe mathematical relationships involving addition and subtraction situations for numbers less than 10.

Example: Using objects verbally describe in a number sentence the joining of a group of three objects with one more object (e.g., $3 + 1 = 4$).

K.2.2 Create, extend, and give the rule for simple patterns with numbers and shapes.

Example: Make a pattern with one square, one circle, one square, one circle, etc. Give a rule for the pattern and tell which shape comes next and justify your choice.

Standard 3

Geometry and Measurement

K.3.1 Identify, describe, sort, compare and classify objects by shape, size, number of vertices and other attributes.

Example: Sort the tagboard shapes in the box into those that have straight sides and those that do not. Make up your own rule for sorting and sort the shapes in a different way.

K.3.2 Identify the positions of objects in space and use the terms inside, outside, between, above, below, near, far, under, over, up, down, behind, in front of, next to, to the left of and to the right of.

Example: Play “Simon Says” with words and motions to put the block inside, outside, above, and below the box.

K.3.3 Make direct comparisons of the length and weight of objects and recognize which object is shorter, longer, taller, lighter or heavier.

Example: Line up pencils side by side from shortest to longest and explain the ordering used.

K.3.4 Identify concepts of time (before/after, shorter/longer, morning, afternoon, evening, today, yesterday, tomorrow, week, month and year).

Example: Tell about something you did yesterday and something you plan to do tomorrow while the teacher records the responses on a chart labeled yesterday and tomorrow.

Process Standards

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
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- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

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- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

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GRADE 1

Standard 1 Number Sense and Computation

1.1.1 Count, read, write, order, rename and compare whole numbers to at least 100.

Examples:

- Use a hundreds chart or manipulatives to compare numbers.
- Rename 87 as $50 + 37$, and then rename in another way.

1.1.2 Name the number that is one more than or one less than any number to at least 100.

Example: Using a 0-99 chart, and then without a chart, name the number that is one more or one less than 79.

1.1.3 Match the ordinal numbers first, second, third, etc. with an ordered set to at least 10 items.

Example: Name the fifth child from the front of the line.

1.1.4 Show equivalent forms of whole numbers to at least 100 as groups of tens and ones.

Example: Use base ten blocks to model 34 using three longs and 4 units, then 34 units, finally using two longs and 14 ones.

1.1.5 Solve problems involving addition and subtraction by modeling addition of numbers to at least 100 (putting together, increasing) and by modeling the inverse operation of subtraction (taking away, comparing, finding the difference) using objects.

Examples:

- Using objects and a number line show 75 equals 43 plus 32 more.
- Using objects and a number line show the difference between 58 and 21.

1.1.6 Demonstrate fluency with addition facts and the corresponding subtraction facts for totals to at least 20.

Example: Practice the “making tens” strategy by completing the following fact families:

$10 = 6 + 4$	$4 = 10 - 6$
$10 = \underline{\quad} + 6$	$6 = 10 - \underline{\quad}$
$\underline{\quad} + 6 = 10$	$10 - \underline{\quad} = 6$
$\underline{\quad} + 4 = 10$	$\underline{\quad} - 6 = 4$

1.1.7 Pose a question and collect and represent data using pictures or picture graphs to answer the question posed.

Example: Decide on a question, ask your classmates, record their responses and then make a picture graph of the results.

Standard 2

Algebra and Functions

1.2.1 Write and solve equations involving addition.

Example: $7 = \quad + 3$.

1.2.2 Create, extend, and give a rule for number patterns using addition.

Example: Given the number pattern 4, 8, 12 ... tell the next number and explain how you determined the pattern to be followed.

1.2.3 Solve problems using the identity principle for addition and subtraction.

Example: Ben put five apples in a bag. When he got to school he realized the bag had a hole. He recounted the apples and he had five. How many apples did lose through the hole?

Standard 3

Geometry and Measurement

1.3.1 Identify, describe, compare, sort and draw triangles, rectangles, squares and circles in terms of their attributes (position, shape, size and number of vertices). Use simple plane shapes to compose a given shape.

Examples:

- Draw shapes on the sidewalk and play a game jumping to the shape that is called.
- Compare squares and rectangles listing how they are alike and different.

1.3.2 Estimate and measure the length of an object to the nearest inch and centimeter.

Example: Locate items around the classroom that are close to one inch or one centimeter and list them on a t-chart. Measure to see if you chose items close to the correct unit.

1.3.3 Give the value of a collection of pennies, nickels and dimes up to \$1.00.

Example: Play “Memory” with cards that have money amounts and pictures of coins. Collect matches and then tell your partner the value of the money represented.

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- Recognize reasoning and proof as fundamental aspects of mathematics.
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- Select and use various types of reasoning and methods of proof.

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GRADE 2

Standard 1 Number Sense and Computation

2.1.1 Count, read, write, compare, and plot on a number line whole numbers to at least 1000.

Examples:

- Use base ten blocks to model 234 using two flats, three longs and 4 ones, then using twenty-three longs and 4 units and finally 234 units.
- Which is more 850 or 805? Justify your answer.

2.1.2 Count by ones, twos, fives, tens and hundreds to at least 1000, and show the number that is ten more or ten less than any number 10 through 90.

Example: Complete a chart with missing numbers and then tell what is ten more or ten less than any number given.

81	82	83	84	85	86	87		89	
	92			95					100
101								109	

2.1.3 Match the ordinal numbers, first, second, third, etc. with an ordered set of at least 100 items.

Example: Given an alphabet chart name the seventeenth letter in the alphabet.

2.1.4 Use words, models, standard form and expanded form to represent place value and to show equivalent forms of whole numbers up to at least 1,000 as groups of hundreds, tens and ones.

Example: Using base ten blocks and a place value mat show at least two different ways that you could model 57. Explain why your two models represent the same number.

2.1.5 Identify numbers as even or odd by placing that number of objects in two groups of the same size and recognizing that for even numbers no object will be left over and for odd numbers one object will be left over.

Example: Two children each pick an object from the pile of 18 objects and each place it in a pile they start. They continue picking objects and placing them in their own pile. After all of the objects have been picked and placed, count to see if each pile has the same number of objects.

2.1.6 Solve problems involving addition and subtraction of whole numbers less than 1000 fluently using a standard algorithmic approach and show the inverse relationship between addition and subtraction.

Examples:

- The students wanted to know the total number of pencils they had remaining from their supply box. They worked in small groups and found that group one had 54 pencils, group two had 37 pencils and group three had 15 pencils. How many pencils did the class have? Explain how you found the answer.
- Their teacher said she had ordered a box of 500 pencils at the beginning of the year. How many pencils should still be in the box? Explain how you found the number of pencils.

2.1.7 Compare data displayed in tables and picture graphs within the table or graph and with data on other tables and graphs to address a single question.

Example: Write a statement about the most and least favorite pets from your class and the class next door.

Standard 2

Algebra and Functions

2.2.1 Write equations to solve single and multi-step addition and subtraction word problems.

Example: You and a friend have collected 352 cans to recycle. You collected 201 cans. Write an equation to show the relationship between the numbers and find how many cans your friend collected.

2.2.2 Create, extend, and give a rule for number patterns using addition and subtraction.

Example: Find the next number in the sequence 101, 99, 97, 95 ... and tell how you found the answer.

2.2.3 Show that the order in which two numbers are added [commutative property] and how the numbers are grouped in addition [associative property] will not change the sum. Use these properties can be used together to show that numbers can be added in any order.

Example: Add the numbers 5, 17, and 13 in this order. Now add them in the order 17, 13, and 5. Show that the results are the same.

Standard 3

Geometry and Measurement

2.3.1 Recognize, identify and describe attributes of common shapes and solids (e.g., the size and type of shape, the two-dimensional faces of three-dimensional figures, the number of sides, edges and vertices; and location in space).

Examples:

- Describe the difference between a circle and a triangle using the terms, sides, edges, and vertices.

- Describe the difference between a cube and a sphere using the terms, sides, edges, and vertices.

2.3.2 Identify and draw congruent two-dimensional shapes in any position. Describe and compare properties of simple and compound figures composed of triangles, rectangles, and squares.

Example: Given a shape on a geoboard create a congruent shape somewhere else on the geoboard.

2.3.3 Measure length in standard units (inch, foot, yard) and metric units (centimeter and meter) and select appropriate units to estimate and measure lengths. Use the relationships between the units to express answers in different units. Use units of linear measurements and relationships within a particular system to solve problems.

Example: Estimate how many meter sticks would lie end to end across the classroom and then measure the length of your classroom to the nearest meter and report the result in centimeters also.

2.3.4 Describe relationships of time (seconds in a minute, minutes in an hour, hours in a day, days in week, and days in a year) and tell time on an analog clock to five-minute intervals.

Example:

- How long is your school day in hours?
- How many minutes do you work on math?
- Read an analog clock to help your teacher know when lunch and recess are over.

2.3.5 Find the value of a collection of pennies, nickels, dimes, quarters and dollars.

Example: You empty your bank and you find: 3 pennies, 4 nickels, and 2 dimes. Do you have enough money to buy a \$.50 pencil?

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GRADE 3

Standard 1

Number Sense and Computation

3.1.1 Count, read, write, compare, and plot on a number line whole numbers up to at least 10,000.

Examples:

- Show where 349 appears on the number line.
- Given another number tell if 349 is less than or greater than this number.

3.1.2 Interpret and model fractions as parts of a whole, parts of a group, and points and distances on a number line for numbers less than, equal to, or greater than one.

Examples:

- Shade $\frac{3}{4}$ of a given shape.
- Model $\frac{3}{4}$ of a collection of beans.
- Label $\frac{3}{4}$ on number line that has $\frac{1}{4}$ increments marked.

3.1.3 Compare and order fractions by using models, benchmark fractions, or common numerators or denominators.

Example: Have students fold paper to make halves, fourths, and eighths. Assign to groups of students specific amounts to color such as $\frac{1}{2}$ or $\frac{1}{4}$. Arrange the finished art on the wall from smallest to largest, showing how $\frac{2}{4} = \frac{1}{2}$.

3.1.4 Use words, models, standard form and expanded form to represent place value and to show equivalent forms of whole numbers up to at least 10,000.

Example: Convert among numbers written in words, in standard form, and expanded form. Four hundred ninety-two = 492 = 400 + 90 + 2.

3.1.5. Solve problems involving addition and subtraction of whole numbers fluently using a standard algorithmic approach.

Example: Bob earned \$547 in July and \$568 in August. Bob told his friend he earned \$1,103. Decide whether Bob is right or not and explain how you know.

3.1.6 Represent the concept of multiplication of whole numbers with models as repeated addition, equal-sized groups, arrays, area models, and equal “jumps” on a number line and explain the result of multiplying by zero.

Examples:

- Using base ten blocks make an array of three rows of three blocks.
- Beginning at zero make three hops of three to nine on the number line.
- Show the same amount as three groups of three items and as

3.1.7 Represent the concept of division of whole numbers with models as successive subtraction, partitioning, sharing, and an inverse of multiplication. Show that division by zero is not possible.

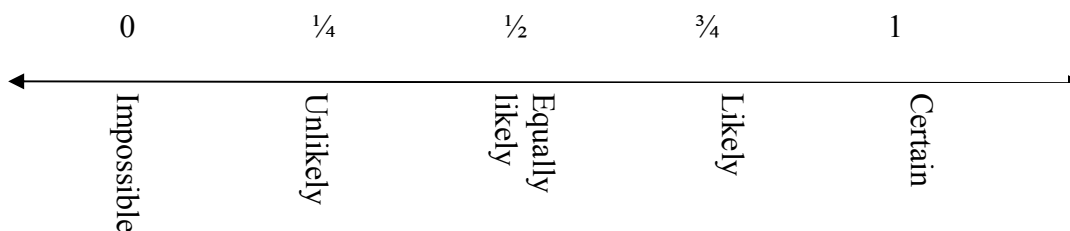
Examples:

- Start with 25 blocks and keep subtracting groups of 5. How many groups of five can be made?
- Start with 25 blocks and find how many rows of five you can make.
- Start with 25 blocks and see how many groups of five you can make.
- Start with 25 blocks and make zero groups.
- Write the equation $25 \div 5 = 5$

3.1.8 Construct and analyze frequency tables and bar graphs from data, including data collected through observations, surveys and experiments. Example: Write a statement about the information displayed in the graph to express a conclusion about the information.

3.1.9 Identify events on a continuum from impossible to unlikely, equally likely, likely or certain. Determine a simple probability in a context using pictures.

- Example: Students will place the letter with the following events on the number line pictured below.



- A. There are six cubes in a jar and two are yellow. How likely are you to pull a yellow cube?
 - B. The probability that you will leave school before midnight.
 - C. The probability a snowman in Indiana will stay in your backyard though the summer.
 - D. The probability that the sun will rise tomorrow.
- Flip a coin and then record the number of times the coin lands heads-up for ten trials, then twenty trials.
 - Spin a three color spinner (red, yellow and green) and display the results for the number of times the spinner lands on a particular color.

Standard 2 Algebra and Functions

3.2.1. Write and solve equations using (=) to show equivalence and use variables to express mathematical relationships involving multiplication.

Example: If one ice cream sandwich costs 20 cents and two cost 40 cents, write the equation that will show how much n ice cream sandwiches cost?

3.2.2 Create, extend, and give a rule for number patterns using multiplication.

Example: Complete the following input/output table. Explain the rule you used to find the output.

Input	Output
5	50
2	20
7	

3.2.3 Solve problems using the identity principle of multiplication.

Example: Use arrays to model 1×5 , 1×10 , 1×3 , 1×12 and then explain what happens whenever you multiply by one.

Standard 3 Geometry and Measurement

3.3.1 Identify angles that are right angles and other angles that are greater than or less than a right angle.

Example: Use the corner of a piece of paper as a right angle finder to search for right angles in the classroom.

3.3.2 Identify, describe and draw points, lines and line segments and use these terms when describing two-dimensional shapes.

Example: Draw a line segment AB that is 2 cm long.

3.3.3 Identify and draw lines of symmetry in geometric shapes and recognize symmetrical shapes in the environment.

Example: Use pencil and paper or a drawing program to draw all the lines of symmetry in a square.

3.3.4 Find the perimeter of polygons.

Example: Measure and then find the perimeter of your desk in centimeters.

3.3.5 Choose and use appropriate units and tools to estimate and measure length and weight. Estimate and measure length to $\frac{1}{4}$ inch, weight in pounds and kilograms, and temperature in Celsius and Fahrenheit selecting appropriate units for the given situation. Use the relationship between the units to express answers in different units.

Examples:

- Estimate and then measure the weight of your book bag in pounds in ounces and pounds.
- Estimate and then read the temperature on a thermometer in degrees Fahrenheit and degrees Celsius.

3.3.6 Using an analog clock tell time to the nearest minute.

Example: If you have to be to school at 8:00 and the car ride is 15 minutes, when do you need to leave for school?

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GRADE 4

Standard 1

Number Sense and Computation

4.1.1 Count, read, write, compare and plot whole numbers using words, models, number lines and expanded form.

Example: Plot the number 980,000 on a number line labeled in increments of 100,000's.

4.1.2 Find equivalent fractions and then use them to compare and order whole numbers and fractions using the symbols for less than (<), equals (=), and greater than (>).

Examples:

- Find three equivalent fractions for $\frac{1}{4}$.
- List from least to greatest: $\frac{7}{8}$, $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{1}{3}$.

4.1.3 Solve problems involving decimals to hundredths.

- Interpret and model decimals as parts of a whole, parts of a group, and points and distances on a number line.
- Use benchmarks (well-known numbers used in meaningful points for comparison) to compare decimals between 0 and 1.0.
- Write decimals as fractions.

Examples:

- Model 0.4 on a ten by ten grid.

- Place 0.5 and 0.8 on a number line marked in tenths.
- Is 0.4 closer to 0 or to 1?
- Write 0.4 as a fraction.

4.1.4 Use words, models, standard form and expanded form to represent place value of decimal numbers to hundredths.

Examples: Explain how you know that 0.4 is equal to 0.40.

4.1.5 Demonstrate fluency with multiplication facts for numbers up to at least 10 and the related division facts. Identify factors of whole numbers and multiplies of whole numbers to 10.

Examples:

- Complete basic facts like $9 \times 4 =$ and $35 \div 7 = ?$ quickly and accurately.
- List all the factors for 36.
- Find the common multiples of four and six.

4.1.6 Solve problems using multiplication of two-digit by single-digit and two-digit numbers fluently using a standard algorithmic approach.

Example: If there are 24 boxes with 36 pencils in each, what is the total number of pencils? Estimate and then compute the product.

4.1.7 Model addition and subtraction of simple fractions.

Example: Using pattern blocks show that if the yellow hexagon equals one whole, which part of the hexagon represents $(1 - 1/3)$?

4.1.8 Construct and analyze line plots. Given a set of data or a graph, describe the distribution of the data using median, range or mode.

Example: Display the following amounts of seconds from your science experiment: 16, 22, 16, 9, 11, 16 and 11 as a line plot. Identify the median, range and mode of your data.

4.1.9 List all the possible outcomes of a given situation or event. Represent the probability of a given outcome using a picture or other graphic.

Example: The Circle Snack Shop has 3 flavors of ice cream- vanilla, chocolate and strawberry. The ice cream can be served in a sugar cone, a waffle cone or in a dish. List all the possible combinations of flavors of ice cream and how it is served.

Standard 2

Algebra and Functions

4.2.1 Write and solve equations with ($=$) to show equivalence and use with variables to express mathematical relationships involving multiplication and division. Plot the points for the corresponding values in the first quadrant.

Examples:

- Find the number n which satisfies the equation $n \times 7 = 42$. Plot the values for n and the solution.
- Solve the equation $n \times 6 = 9 \times 4$

4.2.2 Create, extend, and give a rule for number patterns using multiplication and division and non-numeric growing or repeating patterns.

Example: Given a verbal description create a different representation of a pattern or sequence.

4.2.3 Show that the order in which two numbers are multiplied [commutative property] and how numbers are grouped in multiplication [associative property] will not change the product. Use these properties together to show that numbers can be multiplied in any order.

Example: Draw or build arrays to show that 3 rows of 8 objects use the same number of objects as 8 rows of 3.

4.2.4 Use the distributive property in expressions involving multiplication.

Example: Use the array model to show that $5 \times 7 = (3 \times 7) + (2 \times 7)$.

			3 x 7			

			2 x 7			

Standard 3 Geometry and Measurement

4.3.1 Identify, describe and draw pairs of parallel lines, perpendicular lines, and non-perpendicular intersecting lines using appropriate mathematical tools and technology

Example: Using their arms students will model parallel lines, perpendicular lines and non-perpendicular intersecting lines.

4.3.2 Identify, describe and draw right angles, acute angles, obtuse angles, straight angles and rays using appropriate tools and technology.

Example: Draw two rays that meet to form each of the angles listed above.

4.3.3 Identify shapes that have reflectional and rotational symmetry.

Example: Make a masking tape frame on the floor for a cardboard equilateral triangle. Mark one corner of an equilateral which fits the frame and the corresponding corner on the frame. Rotate the triangle clockwise until it again fits the frame, rotating about what could be called the center of the triangle. Continue to rotate the triangle until it fits again, and continue until the triangle returns to its original position inside the frame. Count how many times the shape will fit in the frame till it returns to its original position. Discuss this triangle as having rotational symmetry of the order of 3.

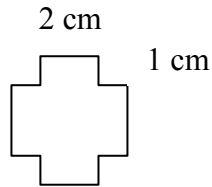
4.3.4 Measure and draw line segments to the nearest eighth-inch and millimeter.

Example: Measure across the face of a nickel to the nearest millimeter.

4.3.5 Develop and use formulas for finding the perimeter and area of rectangles, including squares, using appropriate strategies (e.g. decomposing shapes), tools and units of measure.

Example:

- Measure the length and width of a basketball court and find its area in suitable units.
- Find the area and perimeter of the following shape.



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GRADE 5

Standard 1 Number Sense and Computation

5.1.1 Count, read, write, compare, and plot on a number line decimals to thousandths using words, models and expanded form.

Example: Write the number 198.536 in words.

5.1.2 Compare and order fractions and decimals to thousandths by using the symbols for less than (<), equal to (=), and greater than (>).

Examples:

- Write from the least to the greatest using the correct symbols:
0.5; 0.26; 1.2; 0.008.
- Write from the least to the greatest using the correct symbols:
75%; 33%; 1%.

5.1.3 Identify and explain prime and composite numbers.

Example: List the first five prime counting numbers. Tell why they are prime numbers.

5.1.4 Use words, models, standard form and expanded form to represent place value of decimal numbers to thousandths.

Examples: Explain how you know that 0.005 is less than 0.050

5.1.5 Solve problems involving multiplication and division of whole numbers fluently using a standard algorithmic approach and explain how to treat the remainders in division.

Example: Twenty-six students are going on a field study using the school vans. How many vans are needed if each van holds six students and a driver?

5.1.6 Solve problems involving addition and subtraction of

- decimals, including money;
- fractions, including fractions with different denominators; and
- mixed numbers

using a standard algorithmic approach.

Examples:

- How much money will you have coming home if you start to the store with \$3.00 and you buy popcorn for \$.95 and a drink for \$1.25?
- The cheer squad has $2\frac{1}{4}$ yards of blue fabric, but they need 5 yards to make all the banners needed for the game? How much fabric is still needed?

5.1.7 Solve problems involving the multiplication of fractions using a standard algorithmic approach. Explain the relationship of the product relative to the factors.

Example: After the party there is $\frac{1}{2}$ of a pizza left. If you give $\frac{1}{3}$ of the leftover pizza to your brother, how much of the whole pizza will he get? Explain why the product is smaller than the factors.

5.1.8 Construct and analyze line graphs and double bar graphs from data, including data collected through observations, surveys and experiments.

Example: Complete a line graph to show the changes in temperature over a month.

5.1.9 Perform simple experiments gathering data from a large number of trials and use data from experiments to predict the chance of future outcomes.

Example: Using a spinner with three equal regions, red, yellow, and blue

- Tally the result for 10 spins and then for 30 spins.

- Combine your results with your class and compare results.
- Describe what happens as the number of trials increases.
- Can you determine on which color the next spin will land?
- Can you tell the likelihood that the next spin will land on yellow?

Standard 2
Algebra and Function

5.2.1 Write and evaluate simple algebraic expressions.

Example: Write the equation to represent how many cookies you had if you put three cookies in five bags, plus you had two extra. What is the value if there are four cookies in a bag and still two extras?

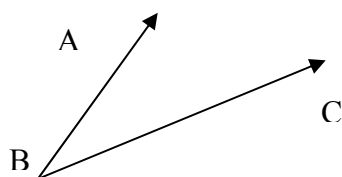
5.2.2 Use two-dimensional coordinate grids to represent points in the first quadrant that fit linear equations and draw the line determined by the points.

Example: Plot the points (3, 1), (6, 2), and (9, 3). Connect the points and tell what you notice.

Standard 3
Geometry and Measurement

5.3.1 Measure angles and describe angles in degrees.

Example: Measure the following angle.



5.3.2 Identify, classify and draw polygons and triangles (equilateral, isosceles, scalene, right, acute and obtuse triangles).

Examples:

- Draw as many different types of quadrilaterals as possible.
- Draw an isosceles right triangle.

5.3.3 Describe the attributes (such as number of edges, vertices, and number of faces) of solids, including cubes, pyramids and cylinders.

Example: Complete the following table.

Solid	Faces	Edges	Vertices
cube	6	12	8
square pyramid		8	5
cylinder	3	0	0
rectangular prism			

5.3.4 Identify and describe using words and pictures, transformations such as reflections, rotations and translations and use this knowledge to design and analyze simple tilings and tessellations.

Example: Using tiles or grid paper create a design that does not have any gaps or holes.

5.3.5 Develop and use the formulas for the perimeter and area of triangles, parallelograms and trapezoids using appropriate units for measures. Find the area of complex shapes by dividing them into basic shapes.

Example: You want to carpet a square room whose sides are 17 feet. You are not going to carpet the area near the fireplace that is 6 feet long and 4 feet wide. What is the area to be carpeted?

5.3.6 Develop and use the formulas for the surface area and volume of rectangular prisms using appropriate units for measures.

Example: Find the surface area and volume of a rectangular box with a length of 30 cm, width of 15 cm, and a height of 10 cm.

Process Standards

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- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication.
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- Recognize and use connections among mathematical ideas.
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- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record, and communicate

mathematical ideas.

- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Round numbers to a specified place value.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
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- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication, and problem solving.
- The challenge is to ensure that technology supports-but is not a substitute for- the development of skills with basic operations, quantitative reasoning, and problem-solving skills.
 - Elementary students should learn how to perform thoroughly the basic arithmetic operations independent of the use of a calculator.
 - The focus must be on learning mathematics, using technology as a tool rather than as an end in itself.

GRADE 6

Standard 1

Number Sense and Computation

6.1.1 Compare, order, and represent on a number line positive and negative integers, fractions, decimals (to hundredths), and mixed numbers.

Example: Find the positions on a number line of 3.56, -2.5, $1\frac{5}{6}$ and -4.

6.1.2 Interpret the absolute value of a number as the distance from zero on a number line, find the absolute value of real numbers, and know that the distance between two numbers on the number line is the absolute value of their difference.

Example: Use a number line to explain the absolute values of -3 and of 7.

6.1.3 Use percents to represent parts of a whole and find the percentage part of a whole.

Example: Draw a circle and shade 45% of it.

6.1.4 Recognize commonly used fractions, decimals, and percents and their equivalents and convert between any two representations of any non-negative rational number without the use of a calculator.

Example: Know that $\frac{1}{3} = 0.333\dots$, $\frac{1}{2} = 0.5$, $\frac{2}{5} = 0.4$, etc.

6.1.5 Solve problems involving addition, subtraction, multiplication and division of integers and represent computation with integers on a number line. Describe the effect of operations with numbers less than zero.

Example: $17 + -4 = ?$; $-8 - 5 = ?$; $3(-6) = ?$; $-12 \div -2 = ?$.

6.1.6 Solve problems involving addition, subtraction, multiplication and division of positive fractions and decimals and explain why a particular operation was used for a given situation.

Example: You want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide. How far from each edge should you place the bar? Explain your method.

Example: Share \$7.25 equally among five people

6.1.7 Interpret ratios, model ratios, and use ratios to show the relative sizes of two quantities.

- Use the notations: a/b , a to b and $a:b$.
- Write equivalent ratios,
- Express a ratio in its simplest form.
- Find the ratio of two given quantities.

Example: A car moving at a constant speed travels 130 miles in 2 hours. Write the ratio of distance to time as a fraction in reduced form.

6.1.8 Recognize proportional relationships and solve problems involving proportional relationships. Find the missing term in a pair of equivalent ratios and find one quantity given the other quantity and their ratio.

Example: Sam made 8 out of 24 free throws. Assuming Sam's success rate continues, find how many free throws he will make with 60 attempts.

6.1.9 Solve simple percent, ratio and proportion problems, including problems involving discounts at sales, interest earned and tips.

Example: In a sale, everything is reduced by 20%. Find the sale price of a shirt whose pre-sale price was \$30.

Standard 2

Algebra and Functions

6.2.1 Write and solve one-step linear equations and inequalities in one variable.

Example: The area of a rectangle is 143 cm^2 and the length is 13 cm. Write and solve an equation to find the width of the rectangle. Describe how you will check to be sure that your answer is correct.

6.2.2 Write and use formulas with up to three variables to solve problems.

Example: You have P dollars in a bank that gives $r\%$ simple interest per year. Write a formula for the amount of interest you will receive in one year. Use the formula to find the amount of interest on \$80 at 6% per year for one year.

6.2.3 Apply the correct order of operations and the properties of real numbers [identity, inverse, commutative, associative and distributive properties] to evaluate numerical expressions, including those that use grouping symbols such as parentheses. Justify each step in the process.

Example: Simplify $3(4 - 1) + 2$. Explain your method.

6.2.4 Identify and graph ordered pairs in all four quadrants of the coordinate plane.

Example: Plot the points $(3, -1)$, $(-6, 2)$ and $(9, -3)$. What do you notice?

6.2.5 Solve problems involving linear functions with integer values. Create a table and graph the resulting ordered pairs of integers on a grid. Look for patterns in how a change in one variable relates to a change in the second variable and write the equation.

Example: A plant is 3 cm high the first time you measure it (on Day 0). Each day after that the plant grows by 2 cm. Write an equation connecting the height and the number of the day. Draw its graph.

Standard 3

Geometry and Measurement

6.3.1 Identify, draw and use the properties of vertical, adjacent, complementary, and supplementary angles, and properties of triangles and quadrilaterals, to solve problems involving a missing angle.

Example: Draw two parallel lines with another line across them. Identify all pairs of supplementary angles.

6.3.2 Recognize that the sum of the interior angles of any triangle is 180° and that the sum of the interior angles of any quadrilateral is 360° . Use this information to solve problems.

Example: Find the size of the third angle of a triangle with angles of 73° and 49° .

6.3.3 Develop and use the formulas for the circumference and area of a circle.

Example: Measure the diameter and circumference of several circular objects. (Use string to find the circumference.) With a calculator divide each circumference by its diameter. What do you notice about the results?

6.3.4 Recognize that real-world measurements are approximations. Identify appropriate instruments and units for a given measurement situation, taking into account the precision of the measurement desired.

Example: A coach is using a wristwatch to measure how fast Carlos and Tyler run a 50 yard dash. The wristwatch displays only hours and minutes. Why is this wristwatch an inappropriate instrument?

6.3.5 Develop and use the formulas for the surface area and volume of a cylinder and find the surface area and volume of three-dimensional objects built from rectangular solids and cylinders.

Example: Find the surface area of a cylindrical can 15 cm high and with a diameter of 8 cm.

Standard 4

Data Analysis and Probability

6.4.1 Construct and analyze circle graphs and stem-and-leaf plots.

Examples:

- Display the following data in a stem-and-leaf plot: \$126; \$118; \$100; \$98; \$95; \$138; \$101; \$116. Write a sentence that summarized the data.
- Would a circle graph be a useful way to display this data?

6.4.2 Choose the appropriate display for a single variable set of data from bar graphs, line graphs, circle graphs and stem-and-leaf plots. Justify the choice of data display.

Example: Sarah is investigating the price of a gallon of milk in fifteen different supermarkets for a school project. Determine whether a bar graph, line graph, circle graph, or a stem-and-leaf plot is the best way to display the data. Explain how the type of display you chose is useful in displaying the data.

6.4.3 Compare the mean, median and mode for a set of data and explain which measure is most appropriate in a given context.

Example: The following numbers represent the number of touchdown passes completed by seven different quarterbacks during a three-year period: 74, 42, 36, 31, 27, 27, 27. Which statistical summary (mean, median, or mode) is most representative of the data? Explain. If an eighth quarterback who completed 90 passes is added to the data, which data would not be affected?

6.4.4 Solve problems involving probability as a measure of chance and verify that the probabilities computed are reasonable.

Example: There are 3 blue, 5 green, and 12 red marbles in a bag. You pick one at random. Write the probability of picking a green marble as a fraction, a decimal, and as a

percent.

6.4.5. Recognize and represent probabilities as ratios, measures of relative frequency, decimals between 0 and 1, and percentages between 0 and 100.

Example: The probability that the Colts will win the Super Bowl next year is .85. Express this probability as a ratio and a percentage.

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GRADE 7

Standard 1

Number Sense and Computation

7.1.1 Read, write, compare and solve problems using whole numbers in scientific notation.

Example: Write 300,000 in scientific notation.

7.1.2 Recognize and compute whole number powers of positive integers.

Example: $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = ?$

7.1.3 Recognize the prime factors of a number and find the prime factorization of whole numbers and write the results using exponents.

Example: $24 = 2 \times 2 \times 2 \times 3 = ?$

7.1.4 Recognize or use prime and composite numbers to solve problems.

Example: A counterexample is an example showing that a statement is not true. Find a counterexample for the statement *All even numbers are composite numbers* and explain how you know it is a counterexample.

7.1.5 Recognize and use the inverse relationship between squaring and finding the square root of a perfect square integer.

Example: Find the area of a square of side 5 cm, and find the side of a square whose area is 144 square cm.

7.1.6 Identify, write, rename, compare and order rational and common irrational numbers and plot them on a number line.

Example: Write in order from smallest to largest: -2 , -2π , $-\sqrt{2}$, $-2\sqrt{2}$.

7.1.7 Solve problems that involve multiplication and division with integers, fractions, decimals and combinations of the four operations.

Example: The temperature one day is 5° . It then falls by 3° each day for 4 days and, after that, rises by 2° each day for 3 days. What is the temperature on the last day? Explain your method.

7.1.8 Solve problems involving percents.

- Find the whole given a part and the percentage
- Find percentage increase or decrease.

Example: The population of a country was 36 million in 1990 and it rose to 41.4 million during the 1990s. What was the percentage increase in the population?

7.1.9 Solve problems involving ratios and proportions.

- Express one quantity as a fraction of another, given their ratio, and vice versa.
- Find how many times one quantity is as large as another, given their ratio, and vice versa.
- Express one quantity as a fraction of another given the two quantities.
- Find the whole, or one part, when a whole is divided into parts in a given ration.
- Solve problems involving two pairs of equivalent ratios.

Example: On a survey of females in an exercise class, 12 out of the 20 females in the class indicated they prefer to exercise in the morning. What percent of the females in the class prefer to exercise in the morning?

Standard 2

Algebra and Functions

7.2.1 Use variables and appropriate operations to write an expression, equation or inequality that represents a verbal description.

Example: Write in symbols the inequality: 5 less than twice the number is greater than 42.

7.2.2 Write and solve two-step linear equations and inequalities in one variable.

Example: Solve the equation $4x - 7 = 12$ and check your answer in the original equation.

7.2.3 Evaluate numerical expressions and simplify algebraic expressions involving rational and irrational numbers.

Example: Simplify $3(4x + 5x - 1) + 2(x + 3)$. Explain each step you take.

7.2.4 Solve an equation or formula with two variables for a particular variable.

Example: Solve the formula $C = 2\pi r$ for r .

7.2.5 Find the slope of a line from its graph and relate the slope of a line to similar triangles.

Example: Draw the graph of $y = 2x - 1$. Choose two points on the graph and divide the change in y -value by the change in x -value. Repeat this for other pairs of points on the graph. What do you notice?

7.2.6 Draw the graph of a line given its slope and one point on the line or two points on the line.

Example: Draw the graph of the equation with slope of 3 and passing through the point with coordinates $(0, -2)$.

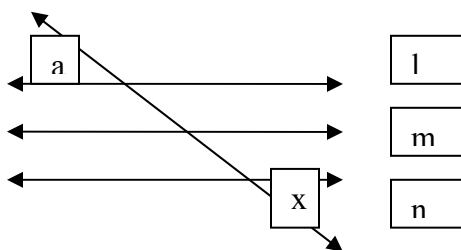
7.2.7 Identify situations that involve proportional relationships, draw graphs representing these situations and recognize that these situations are described by a linear function in the form $y = mx$ where the unit rate m is the slope of the line.

Example: At a ski resort, one of the slopes rises 8 feet vertically for every 48 foot run. The second slope rises 12 feet vertically for every 72 feet of run. Compare the steepness of the two slopes. Which is steeper?

Standard 3 Geometry and Measurement

7.3.1 Identify and use basic properties of angles formed by transversals intersecting pairs of parallel lines.

Example: Find the value of angle x in the figure below if $m\angle a = 30^\circ$



7.3.2 Identify, describe, and use transformations (translations, rotations, reflections and simple compositions of these transformations) to solve problems.

Example: Draw a triangle with vertices $(2, 3)$, $(5, 3)$, and $(2, 7)$. Translate (slide) the triangle so that the vertex $(2, 3)$ moves to $(0, 0)$. Find the new coordinates of the other two vertices.

7.3.3 Draw two-dimensional patterns (nets) for three-dimensional objects, such as right prisms, pyramids, cylinders and cones.

Example: Draw a rectangle and two circles that will fit together to make a cylinder.

7.3.4 Recognize, describe, or extend geometric patterns using tables, graphs, words, or symbols.

Example: Look at the three triangles below. What fraction of each triangle is not shaded? Do you see a pattern? Use the pattern to predict the fraction of the triangle you would NOT shade in the fourth iteration of the triangle. Confirm your prediction and explain.



7.3.5 Identify, describe, and construct similarity relationships and solve problems involving similarity (including similar triangles) and scale drawings by using proportional reasoning.

Example: At a certain time, the shadow of your school building is 36 feet long. At the same time, the shadow of a yardstick held vertically is 4 feet long. What is the height of the school building?

7.3.6 Solve simple problems involving distance, speed and time.

- Understand concepts of speed and average speed.
- Understand the relationship between distance, time and speed.
- Find speed, distance or time given the other two quantities.
- Write speed in different units (km/h, m/s, cm/s, mi/hr, ft/sec).
- Solve simple problems involving speed and average speed.

Examples:

- Find how long an airplane flying at 900 kilometers per hour takes to travel 1,350 kilometers.
- A cheetah can run 120 kilometers per hour for a short time. Give this speed in meters per second.
- Sarah drove 145 miles from Elkhart to Muncie in 2 hours and 45 minutes. Find Sarah's average speed in miles per hour.

Standard 4

Data Analysis and Probability

7.4.1 Create, analyze and interpret data sets in multiple ways using bar graphs, frequency tables, line plots, histograms and circle graphs. Justify the choice of data display.

Example: The students will count the value of the change in their pockets. Use the amounts collected from the class to construct a histogram. Describe the shape of the distribution.

7.4.2 Make predictions from statistical data and use proportions to make estimates about a population based on a sample.

Example: Record the temperature and weather conditions (sunny, cloudy or rainy) at 1 p.m. each day for two weeks. In the third week, use your results to predict the temperature from the weather conditions.

7.4.3 Describe how additional data, particularly outliers, added to a data set may affect the mean, median and mode.

Example: You measure the heights of the students in your grade on a day when the basketball team is playing an away game. Later you measure the players on the team and include them in your data. What kind of effect will including the team have on the mean, median and mode? Explain your answer.

7.4.4 Analyze data displays, including ways that they can be misleading. Analyze ways in which the wording of questions can influence survey results.

Example: A company displays a bar graph of company's sales that suggests sales have more than doubled since last year. Upon analyzing the graph, you notice that sales have in fact increased from \$5.5 million to \$6.2 million. Explain how the company may have used the graph to suggest that sales doubled?

7.4.5. Understand that when all outcomes of an experiment are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event occurs. Use theoretical probability and proportions to make approximate predictions.

Example: The weather forecast says that the probability of rain today is 0.3. What is the probability that it will not rain?

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GRADE 8

Standard 1

Number Sense and Computation

8.1.1 Interpret calculator or computer displays of numbers given in scientific notation and read, write, compare and solve problems using decimals in scientific notation.

Example: Write 0.00357 in scientific notation.

8.1.2 Recognize positive integer powers as repeated multiplication and negative integer powers as repeated division or multiplication by the multiplicative inverse.

Example: Write 2^{-3} as a fraction.

8.1.3 Use the laws of exponents for integer exponents and evaluate expressions with negative integer exponents.

Example: Write $2^2 \times 2^3$ as (2×2) $(2 \times 2 \times 2)$ and then as a single power of 2. Explain what you are doing. Evaluate $6^{-3} / 6^2$. Explain your process.

8.1.4 Identify, compare and order irrational numbers.

Example: Estimate the square root of $\sqrt{18}$ to the nearest tenth. Plot the square root on a number line.

8.1.5 Calculate square roots of perfect squares, estimate square roots of numbers less than 1,000, and use the inverse relationship between squares and square roots.

Example: Explain how you can find the length of the hypotenuse of a right triangle with legs that measure 5 cm and 12 cm.

8.1.6 Solve percent, ratio and proportion problems.

- Find average rates.
- Express one quantity as a percentage of another.
- Compare two quantities by percentage.
- Use percentages greater than 100%.
- Increase or decrease a quantity by a given percentage.
- Find the original amount for a given percentage increase or decrease.
- Solve problems involving percents, ratios and proportions.
- Solve problems involving simple and compound interest.

Example: You leave \$100 in each of three bank accounts paying 5% interest per year. One account pays simple interest, one pays interest compounded annually, and the third pays interest compounded quarterly. Use a spreadsheet to find the amount of money in each account after one year, two years, three years, ten years and twenty years. Compare the results in the three accounts and explain how compounding affects the balance in each account.

Standard 2

Algebra and Functions

8.2.1 Write and solve linear equations and inequalities, interpret the solution or solutions in their context, and verify the reasonableness of the results.

Example: As a salesperson, you are paid \$50 per week plus \$3 per sale. This week you want your pay to be at least \$100. Write an inequality for the number of sales you need to make, solve it, and check that your answer is reasonable.

8.2.2 Solve equations and formulas for a specified variable.

Example: Solve $5(x + 2) = 10 + 5$

8.2.3 Simplify algebraic expressions involving powers.

Example: Expand $3(2x + 5)^2$

8.2.4 Identify and graph linear functions and identify lines with positive and negative slope.

Example: Draw the graphs of $y = 2x - 1$, $y = 3x - 1$, $y = -2x - 1$, and $y = -3x - 1$. Find the slope of each graph. What do you notice?

8.2.5 Find the slope of a linear function given the equation and write the equation of a line given the slope and any point on the line.

Example: Write an equation of the line with slope 2 and containing the point $(1, -2)$. What is the y -intercept for this line?

8.2.6 Translate among tables, equations, verbal expressions and graphs of linear functions and recognize that in $(y = mx + b)$ that m is the rate of change and b is the vertical intercept of the graph.

Example: Write a formula for the perimeter of a square as a function of its side length. Construct a table of values for this function. Draw the graph of this function.

8.2.7 Identify functions as linear or nonlinear and contrast their characteristics from tables, graphs and equations.

Example: Graph $y = x^2 + 2$ and $y = x + 2$ on the same coordinate plane. How does the graph of $y = x^2 + 2$ differ from the graph of $y = x + 2$?

8.2.8 Use linear functions and linear equations to represent, analyze and solve problems.

Example: Corey is hosting a birthday party for a friend at a park shelter. The cost to rent the shelter is \$50 plus \$2 per person. Write a function to represent the situation. Find the total cost if 35 people attend.

Standard 3 Geometry and Measurement

8.3.1 Perform basic compass and straight edge constructions: angle and segment bisectors, copies of segments and angles, and perpendicular segments. Describe and justify the constructions.

Example: Explain the procedures used to construct the three angle bisectors of a triangle

8.3.2 Identify, define, and describe properties of three-dimensional geometric objects, describe how two or more figures intersect in a plane or in space, and visualize or describe the cross section of a solid.

Example: Find two lines in your classroom that are not parallel, yet do not meet.

8.3.3 Explain why the Pythagorean Theorem is valid using a variety of methods and use the Pythagorean Theorem and its converse to calculate lengths of line segments.

Example: A square piece of glass 7 feet on each side must be delivered through a doorway. Can the glass fit through the doorway that is 3 feet wide and 6.5 feet tall? Explain your thinking. Make a scale drawing on grid paper to solve the problem.

8.3.4 Solve simple problems involving rates and other derived measurements, including problems involving speed, uniform speed, average speed and density, by applying the concept of proportionality to measurement in different contexts. Express measurements in a given unit or in terms of other units of the same type.

Example: A car travels at 60 mph for 20 minutes and then at 48 miles an hour for 10 minutes. What is the average speed in miles per hour for this trip? Explain your answer.

8.3.5 Use scale factors to find the area and volume of similar figures.

Example: Calculate the volume and surface area of cubes with side 1 cm, 2 cm, 3 cm, etc. Make a table of your results and describe any patterns in the table.

8.3.6 Find and use the surface area and volume of cones, spheres and pyramids.

Example: A prism has its base a right triangle with the shorter sides of length 6 and 8 feet. Its height is 14 feet and non-triangular faces are rectangles. Find the surface area of the prism.

8.3.7 Estimate and compute the area of irregular two-dimensional shapes and the volume of irregular three-dimensional objects by breaking them down into more basic geometric objects.

Example: Find the volume of a doghouse that has a rectangular space that is 3 ft long by 2 ft wide by 5 ft high and has a triangular roof that is 1.5 ft higher than the walls of the house.

8.3.8 Solve problems involving conversions within the same measurement system and estimate the measure of an object in one system given the measure of that object in another system and the approximate conversion factor.

Example: The area of a hall is 40 square yards. What is the area in square feet?

Standard 4

Data Analysis and Probability

8.4.1 Identify claims based on statistical data and, in simple cases, evaluate the reasonableness of the claims. Design a study to investigate the claim.

Example: A study shows that teenagers who use a certain brand of toothpaste have fewer cavities than those using other brands. Describe how you can test this claim in your school.

8.4.2 Identify different methods of selecting samples, analyzing the strengths and weaknesses of each method and the possible bias in a sample or display.

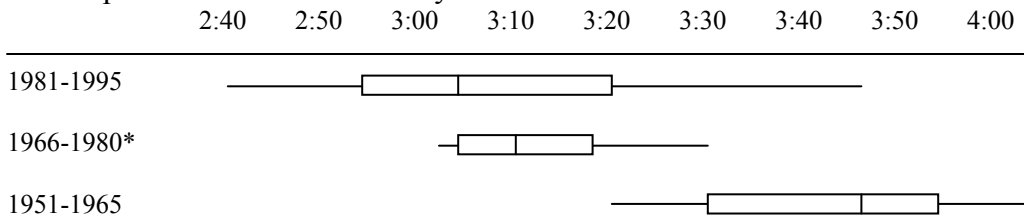
Example: Describe possible bias in the following survey: A local television station has a daily call-in poll. Viewers of the morning and noon newscasts are asked to call one telephone number to answer “yes” and a different telephone number to answer “no.” The results are reported on the five-o’clock newscast.

8.4.3 Use mean, median, mode, upper and lower quartiles and range to compare data sets. Organize and display data to highlight important features such as the range and how the data is spread around a central value. Investigate what happens to the display when some of the data values are changed.

Example: Arrange a set of test scores in increasing order and find the lowest and highest scores, the median and the upper and lower quartiles.

8.4.4 Analyze, interpret and display data in box-and-whisker plots.

Example: The box-and-whisker plots below show winning times (hours: minutes) for the Indianapolis 500 race in selected years:



*Except 1967, 1973, 1975, and 1976.

In the years from 1951-1965, the slowest time was 3 h 57 minutes. Explain how the slowest time changed through the years 1951-1995. How did winning times change during that period? How did the median times change in the same period?

8.4.5 Display two-variable data in scatter plots and describe how the data points are distributed. If the pattern appears to be linear, draw a line that appears to best fit the data and write the equation of that line.

Example: Survey some of the students at each grade level in your school, asking them how much time they spend on homework. Plot the grade level and time of each student as a point (grade, time) on a scatter diagram. Describe and justify any relationship between grade and time spent on homework.

8.4.6 Describe and apply the addition rule for probabilities for simple events that are mutually exclusive and for simple events that are not.

Example: Amy and Bill were each asked to roll two six-sided dice and add the numbers shown. Amy wins if the sum is odd. Bill wins if the sum is six, seven, or eight. Who is more likely to win? Show your work.

8.4.7 Compute probabilities of events from simple experiments with equally probable outcomes, using such methods as organized list, tree diagrams and area models.

Example: A six-sided die is rolled and a spinner with equal sections A, C, D and E is spun. Find the probability of rolling a 3 and spinning a vowel.

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Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication
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- Use representations to model and interpret physical, social, and mathematical phenomena

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.

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- Determine appropriate accuracy and precision of measurement in problem situations.
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- Recognize when the numbers involved in a computation allow for a mental computation strategy.

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ALGEBRA I

Standard 1

Relations and Functions

A1.1.1 Determine whether a relation represented by a table, graph, words or equation is a function or not a function and translate among tables, graphs, words and equations.

Example: For a square of side x , the area y is given by $y = x^2$. Is y a function of x ? Is x a function of y ? Answer the same questions for $y = x^2$ if you are told that this holds for negative as well as positive values of x .

A1.1.2 Identify the domain and range of relations represented by tables, graphs, words, and equations.

Example: What is the largest domain for x when $y = x^2$? What is the range of y in this case?

Standard 2

Linear Functions, Equations and Inequalities

A1.2.1 Translate among various representations of linear functions including tables, graphs, words and equations.

Example: Use a spreadsheet to create a table of values for the function $y = -\frac{1}{2}x + 5$.

Graph the function.

A1.2.2 Graph linear equations and show that they have constant rates of change.

Example: Kathy borrowed \$80 from her mother and plans to pay her mother \$10 per week until the debt is paid. The equation for the amount of money Kathy owes her mother is $y = 80 - 10x$, where x is the number of weeks after the loan. Graph the equation. What does the slope of the graph represent?

A1.2.3 Determine the slope, x -intercept, and y -intercept of a line given its graph, its equation, or two points on the line and determine the equation of a line given sufficient information.

Example: Find the slope and y -intercept of the line $4x + 6y = 12$.

A1.2.4 Write, interpret, and translate among equivalent forms of equations for linear functions (slope-intercept, point-slope, and standard), recognizing that equivalent forms reveal more or less information about a given situation.

Example: Write the equation of the line $4x + 6y = 12$ in slope-intercept form. What is the slope of this line? Explain your answer.

A1.2.5 Solve problems that can be modeled using linear equations and inequalities, interpret the solutions, and determine whether the solutions are reasonable.

Example: As your family is traveling along an interstate, you note the distance traveled every 5 minutes. The distance is approximately the same. You graph the distance traveled as a function of time, assuming that what was found for five-minute time intervals holds for all time intervals up to two hours. Draw a linear graph representing this trip. Predict the time of a journey of 50 miles. What does the slope of the graph represent?

A1.2.6 Graph a linear inequality in two variables.

Example: Draw the graph of the inequality $6x + 8y \geq 24$ on a coordinate plane.

Standard 3

Pairs of Linear Equations and Inequalities

A1.3.1 Understand the relationship between a solution of a pair of linear equations in two variables and the graphs of the corresponding lines and solve pairs of linear equations in two variables by graphing, substitution or elimination.

Example: Solve the system of equations: $2y + x = 10$ and $x = y + 3$. Graph the two lines, labeling the point of intersection.

A1.3.2 Graph the solution set for a pair of linear inequalities in two variables with and without technology and use the graph to find the solution set.

Example: Graph the inequalities $y \leq 4$ and $x + y \leq 5$. Shade the region where both inequalities are true.

A1.3.3 Solve problems that can be modeled using pairs of linear equations in two variables, interpret the solutions, and determine whether the solutions are reasonable.

Example: The income a company makes from a certain product can be represented by the equation $y = 10.5x$ and the expenses for that product can be represented by the equation y

$= 5.25x + 10,500$, where x is the number of units of the product sold and y is the number of dollars. How many units of the product must be sold for the company to reach the break-even point?

Standard 4 **Polynomials**

A1.4.1 Use the laws of exponents for variables with exponents and multiply, divide, and find powers of variables with exponents.

Example: Simplify a^2b^6 (a^3), $(n + 2)(n - 2)$, and $(n + 2)^2$.

A1.4.2 Add, subtract and multiply polynomials and divide polynomials by monomials.

Example: Subtract $(4x^2 - 7x + 2) - (x^2 + 4x - 5)$, multiply $(n + 2)(4n - 5)$, and divide $4x^3y^2 + 8xy^4 - 6x^2y^5$ by $2xy^2$.

A.1.4.3 Factor common terms from polynomials and factor quadratic expressions.

Example: Factor $4ax + 3ay + 4bx + 3by$, $2x^2 - 7x + 3$, and $9x^2 - 4$.

Standard 5 **Quadratic Equations and Functions**

A1.5.1 Graph quadratic functions.

Example: Draw the graph of $y = x^2 - 3x + 2$. Using a graphing calculator or a spreadsheet (to generate a data set), display the graph to check your work.

A1.5.2 Solve quadratic equations in the real number system with real number solutions by factoring, by completing the square, and by using the quadratic formula.

Example: Solve the equation $x^2 - x + 2 = 0$ in three ways, by factoring the polynomial, by the quadratic formula and by completing the square. Derive the general quadratic formula by applying the method of completing the square— to $ax^2 + bx + c = 0$.

A1.5.3 Solve problems that can be modeled using quadratic equations, interpret the solutions, and determine whether the solutions are reasonable.

Example: A ball falls so that its distance above the ground can be modeled by the equation $s = 100 - 16t^2$, where s is the distance above the ground in feet and t is the time in seconds. According to this model, at what time does the ball hit the ground?

A1.5.4 Analyze and describe the relationships among the solutions of a quadratic equation, the zeros of a quadratic function, the x -intercepts of the graph of a quadratic function, and the factors of a quadratic expression.

Example: A graphing calculator can be used to solve $3x^2 - 5x - 1 = 0$ to the nearest tenth. Justify using the x -intercepts of $y = 3x^2 - 5x - 1$ as the solutions of the equation.

A1.5.5 Sketch and interpret linear and non-linear graphs representing given situations and identify independent and dependent variables.

Example: The height (h) above water of a diver t seconds after she steps off a platform 100 feet height is given by the formula $h = 100 - 16t$. Graph the function.

Standard 6

Rational and Radical Expressions and Equations

A1.6.1 Add, subtract, multiply, divide, reduce, and evaluate rational expressions with polynomial denominators. Simplify rational expressions with linear and quadratic denominators, including denominators with negative exponents.

Example: Simplify $\frac{x^2-4}{x^5} \div \frac{x^3-8}{x^8}$

A1.6.2 Solve equations involving rational and common irrational expressions.

Example: Solve $\frac{x+5}{4} = \frac{3x+5}{7}$ and $\frac{8}{x} + \frac{28}{x^2-4} = \frac{7}{x-2}$.

A1.6.3 Simplify radical expressions involving square roots.

Example: Assuming that x and y represent non-negative real numbers, simplify $\sqrt{18xy^2}$.

A1.6.4 Solve equations that contain radical expressions on only one side of the equation and identify extraneous roots when they occur.

Example: Solve the equation $\sqrt{x+6} = x$.

Standard 7

Data Analysis

A1.7.1 Organize and display data using appropriate methods to detect patterns and departures from patterns. Summarize the data using measures of center (mean, median) and spread (range, percentiles, variance, standard deviation). Compare data sets using graphs and summary statistics.

Example: Design and conduct a survey about the number of electronic games owned by girls and boys in your school. Organize and display the results of your survey in an appropriate graph. Describe the technique you used to get a random sample. Find the mean, median and mode of your survey data. Which of these gives a useful summary of the data?

A1.7.2 Distinguish between random and non-random sampling methods, identify possible sources of bias in sampling, describe how such bias can be controlled and reduced, evaluate the characteristics of a good survey and well-designed experiment, design simple experiments or investigations to collect data to answer questions of interest, and make inferences from sample results.

Example: To determine what type of videos their customers liked, Drake Video surveyed every tenth person to walk in their store. Describe the sampling method used by Drake Video. Is it an unbiased sampling? Explain your answer.

A1.7.3 Evaluate reports based on data published in the media by considering the source of the data, the design of the study, the way the data are analyzed and displayed and whether the report confuses correlation with causation.

Example: Find an example of a graph in a newspaper or magazine that could be considered misleading. Explain why the graph could be misleading.

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ALGEBRA II

Standard 1 Functions

A2.1.1 Find the zeros, domain, and range of a function.

Example: $f(x) = x^3 - 3x^2 - x + 3$. What is the domain and range of this function?

A2.1.2 Use and interpret function notation, including evaluation of functions represented by tables, graphs, words, equations or a set of ordered pairs.

Example: Given the function, f , below, find the indicated value, if possible.

- represented by the set of ordered pairs $\{(3,5), (2,-3), (1,7), (0,2)\}$, find the value of $f(1)$;
- where $f(x) = \sqrt{4-x}$, find the value of $f(2)$ and $f(8)$

A2.1.3 Recognize and describe the relationships among the solutions of an equation, the zeros of a function, the x -intercepts of a graph, and the factors of a polynomial expression.

Example: Solve the equation $x^4 + x^3 - 7x^2 - x + 6 = 0$, given that $x - 2$ and $x + 3$ are factors of $x^4 + x^3 - 7x^2 - x + 6$.

Standard 2 Linear and Absolute Value Equations, Inequalities and Functions

A2.2.1 Solve systems of linear equations and inequalities in three variables by substitution and elimination.

Example: Solve the system of equations: $x - 2y + 3z = 5$, $x + 3z = 11$, $5y - 6z = 9$.

A2.2.2 Solve problems that can be modeled using systems of linear equations up to three variables, interpret the solutions, and determine whether the solutions are reasonable.

Example: Each week you can work no more than 20 hours all together at the local bookstore and the drugstore. You prefer the bookstore and want to work at least 10 more hours there than at the drugstore. Draw a graph to show the possible combinations of hours that you could work.

A2.2.3 Graph piecewise-defined functions.

Example: Graph the function $f(x) = \begin{cases} x + 2 & \text{if } x \leq 0 \\ 3x - 1 & \text{if } x > 0 \end{cases}$.

A2.2.4 Solve equations and inequalities involving the absolute value of a linear function.

Example: Solve the inequality $|x - 5| \geq 8$ and graph the solution.

Standard 3

Quadratic Equations and Functions

A2.3.1 Define, add, subtract, multiply and divide complex numbers. Represent complex numbers, and the addition, subtraction and absolute value of complex numbers, in the complex plane.

Example: Let $z = 7 - 4i$ and $w = 10 + 6i$. Graph z , w and $z + w$. Prove that the number 0, z , w and $z + w$ are the vertices of a parallelogram on the complex plane.

A2.3.2 Solve quadratic equations in the complex number system.

Example: Solve $x^2 - 2x + 5 = 0$ over the complex numbers.

A2.3.3 Analyze, describe, and sketch graphs of quadratic functions including the lines of symmetry.

Example: Find the zeros for $y = x^2 - 4$. If $y = x^2 - 4$ has a maximum or minimum value, give the ordered pair corresponding to the maximum or minimum point.

A2.3.4 Determine how the graph of a parabola changes if a , b , and c changes in the equation $y = a(x - b)^2 + c$. Find an equation for a parabola given sufficient information.

Example: Write the equation of the parabola with vertex (3, 6) and y -intercept 2 in vertex form.

A2.3.5 Solve problems that can be modeled using quadratic equations and functions, interpret the solutions, and determine whether the solutions are reasonable.

Examples:

- Write the equation of the parabola with vertex (3, 6) and y -intercept 2 in vertex form.
- Describe similarities and differences in the graphs of $y = 2x$ and $y = 2(x - 1) + 3$ without first graphing the equations.

Standard 4
Polynomial Expressions, Equations and Functions

A2.4.1 Analyze, describe, and sketch graphs of polynomial functions by examining intercepts, zeros, domain and range, and end behavior.

Example: Determine by inspection the end behavior of the graph of the function.

$$f(x) = -2x^3 + x^2 + 4x - 5.$$

A2.4.2 Use the binomial theorem to expand binomial expressions raised to positive integer powers.

Example: Expand $(x + 2)^4$.

A2.4.3 Perform arithmetic operations, including long division and division with remainders, on polynomials by others of equal or lower degree.

Example: Divide $2x^3 - 3x^2 + x - 6$ by $x^2 + 2$.

A2.4.4 Factor polynomials completely and solve polynomial equations by factoring.

Example: Solve $x^3 + 27 = 0$ by factoring.

A2.4.5 Use graphing technology to find approximate solutions for polynomial equations.

Example: Approximate the solution(s) of $x^4 - 3x^3 + 2x - 7 = 0$ to the nearest tenth.

A2.4.6 Solve problems that can be represented or modeled using polynomial equations, interpret the solutions, and determine whether the solutions are reasonable.

Example: You want to make an open-top box with a volume of 500 cubic inches from a piece of cardboard that is 25 inches by 15 inches by cutting squares from the corners and folding up the sides. Then use your results to give formula for the volume of the box."

A2.4.7 Find a polynomial function of lowest degree with real coefficients given its roots and use the relationship between solutions of an equation, zeros of a function, x -intercepts of a graph and factors of a polynomial expression to solve problems.

Example: Write an equation that has solutions $x = 2$, $x = 5i$ and $x = -5i$.

Standard 5
Rational and Radical Expressions, Equations and Functions

A2.5.1 Analyze, describe, and sketch graphs of rational functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.

Example: Find the equations of the horizontal and vertical asymptotes of the

function $f(x) = \frac{x+1}{x+5}$.

A2.5.2 Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions, including expressions with negative exponents in the denominator.

Example: Simplify $\frac{x^2-4}{x^5} \div \frac{x^3-8}{x^8}$.

A2.5.3 Understand the properties of rational exponents and use the properties to simplify, multiply, divide, and find powers of expressions containing negative and fractional exponents. Relate expressions containing rational exponents to the corresponding radical expressions.

Example: Write the expression $\left(x^{\frac{1}{2}}y^{\frac{-2}{3}}\right)^6$ in simplest form. Assume all variables are positive

A2.5.4 Analyze, describe, and sketch graphs of square root and cube root functions by examining intercepts, zeros, domain and range, and end behavior.

Example: Graph the function $y = \sqrt{x} + 7$ and find the domain and range.

A2.5.5 Solve equations that contain radical expressions and identify extraneous roots when they occur.

Example: Solve the equation $x = \sqrt{x} + 2$.

A2.5.6 Solve problems that can be modeled using equations involving rational and radical functions, including problems of direct and inverse variation. Interpret the solutions, and determine whether the solutions are reasonable.

Example: Two students, working independently, can complete a particular job in 20 minutes and 30 minutes, respectively. How long will it take to complete the job if they work together at the same rate as when doing the job alone?

Standard 6 Exponential and Logarithmic Functions

A2.6.1 Analyze, describe, and sketch graphs of exponential functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.

Example: Draw the graphs of the functions $y = 2^x$ and $y = 2^{-x}$.

A2.6.2 Know that the inverse of an exponential function is a logarithm, use laws of exponents to derive laws of logarithms, and use the inverse relationship between exponential functions and logarithms and the laws of logarithms to solve problems.

Example: If you know that $\log(2) = a$ and $\log(3) = b$, find $\log(36)$ in terms of a and b .

A2.6.3 Solve exponential and logarithmic equations.

Example: Solve the equation $\log_2 x = 5$.

A2.6.4 Solve problems that can be modeled using exponential and logarithmic equations, interpret the solutions, and determine whether the solutions are reasonable using technology as appropriate.

Example: The population of a certain country can be modeled by the equation $P(t) = 50e^{0.02t}$, where P is the population in millions and t is the number of years after 1900. Find when the population is 100 million, 200 million and 400 million. What do you notice about these time periods?

Standard 7

Sequences and Series

A2.7.1 Write the recursive formula for arithmetic and geometric sequences and find specific terms of arithmetic and geometric sequences.

Example: Find the tenth term of the arithmetic sequence 3, 7, 11, 15 ...

A2.7.2 Write the formula for the general term for arithmetic and geometric sequences and make connections to linear and exponential functions.

Example: Write the formula for the general term of the geometric sequence 2, 6, 18, 54, 162...

A2.7.3 Find partial sums of arithmetic and geometric series.

Example: In the last example, find the sum of the first 10 terms.

A2.7.4 Solve problems involving applications that can be modeled using sequences and finite arithmetic and geometric series, interpret the solutions, and determine whether the solutions are reasonable using spreadsheets as appropriate.

Example: A restaurant has square tables which seat four people. When two tables are placed together, six people can be seated. If 20 square tables are placed together to form one long table, how many people can be seated?

Standard 8

Data Analysis and Probability

A2.8.1 Use the relative frequency of a specified outcome of an event to estimate the probability of the outcome and apply the law of large numbers in simple examples.

Example: There are 15 students who work in a bookstore. If the bookstore needs 3 people to operate, how many consecutive days could the bookstore operate without the same three people working twice?

A2.8.2 Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events.

Example: When a die is rolled 3 times, what is the probability of obtaining a 6, followed by an even number, followed by a 4?

A2.8.3 Know and apply the characteristics of the normal distribution.

- Identify settings in which the normal distribution may be useful.

- Determine whether a set of data appears to be uniform, skewed or normally distributed.
- Use the empirical rule to find probabilities that an event will occur in a specific interval that can be described in terms of one, two or three standard deviations about the mean.

Example: Math SAT scores are normally distributed with mean 500 and standard deviation 100. What is the probability that a randomly selected student's SAT score is greater than 600?

A2.8.4 Use permutations, combinations, and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.

Example: There are 5 students who work in a bookshop. If the bookshop needs 3 people to operate, how many days straight could the bookstore operate without the same group of students working twice?

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GEOMETRY

Standard 1

Points, Lines, Angles and Planes

G.1.1 Find the length of line segments in one- or two-dimensional coordinate systems, the slopes of line segments in two-dimensional coordinate systems, and find the point that is a given fractional distance from one end of the segment to another.

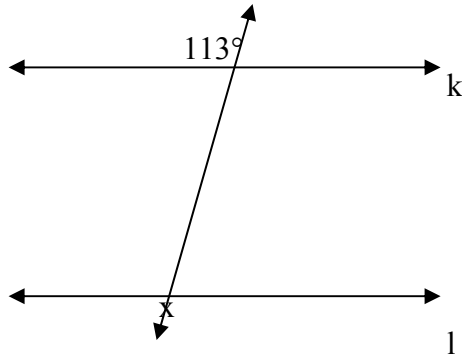
Example: Find the length of the line segment joining A(3,8) and B(9,0). Find the midpoint of this segment and the point that is $\frac{1}{3}$ of the way from A to B.

G.1.2 Construct congruent segments and angles, angle bisectors, perpendicular bisectors, and parallel and perpendicular lines using appropriate geometric construction tools, explaining and justifying the process used.

Example: Construct the perpendicular bisector of a given line segment, justifying each step of the process.

G.1.3 Recognize, use, and justify the relationships between special angles created by parallel lines and transversals.

Example: In the diagram, the lines k and l are parallel. What is the measure of angle x ? Explain your answer.



G.1.4 Identify and apply properties of and theorems about parallel and perpendicular lines, and write equations of parallel and perpendicular lines, and develop simple geometric proofs involving parallel and perpendicular lines.

Example: Find an equation of a line perpendicular to $y = 4x - 2$ which contains the point $(4, 1)$.

G.1.5 Identify, justify and apply properties of planes.

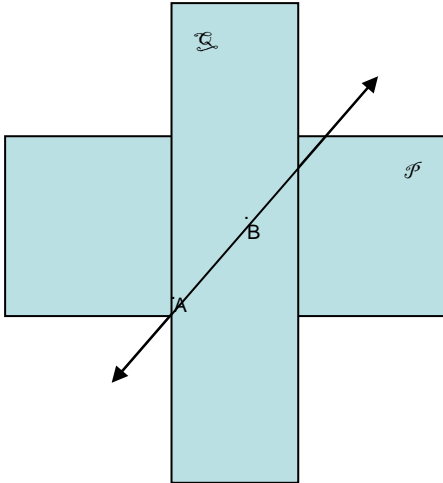
Example: Describe the intersection of plane R with parallel planes S and T .

G.1.6 Represent geometric objects and figures algebraically using coordinates, use algebra to solve geometric problems, and develop simple coordinate proofs involving geometric objects in the coordinate plane.

Example: Draw a triangle with vertices at $(1, 3)$, $(2, 5)$, and $(6, 1)$. Draw another triangle with vertices $(-3, -1)$, $(-2, 1)$, and $(2, -3)$. Are these triangles the same shape and size? Justify your answer.

G.1.7 Describe the intersection of two or more geometric figures in the plane.

Example: Write a description for the figure below



Standard 2
Polygons

General

G.2.1 Find and use the sum of the measures of interior and exterior angles of convex polygons, justifying the method used.

Example: Calculate the measure of one interior angle of a regular octagon. Explain your method.

G.2.2 Identify types of symmetry (line, point, rotational, self-congruences) of polygons.

Example: ABCD is a rhombus. Identify and describe its reflection and rotation symmetry.

G.2.3 Solve problems involving congruent and similar polygons.

Example: In the figures below, Figure A \cong Figure B. Justify this statement: $\angle x \cong \angle y$.

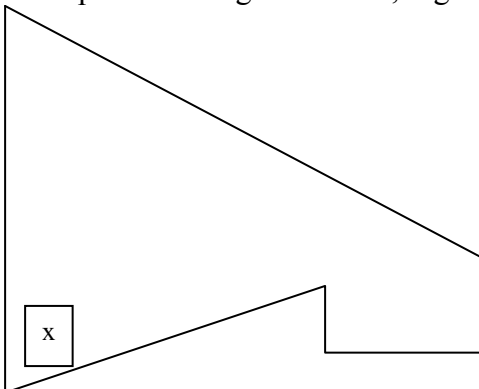


Figure A

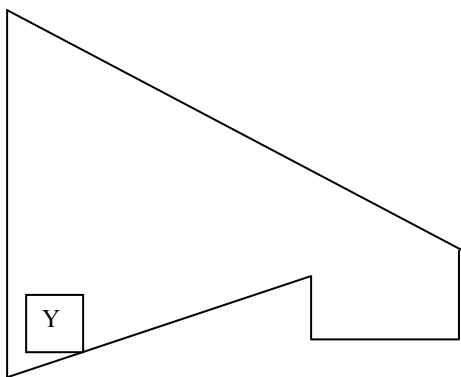


Figure B

G.2.4 Predict and describe the results of translations, reflections, and rotations on polygons and describe a motion or series of motions that will show that two shapes are congruent.

Example: Use a drawing program to create quadrilaterals, regular hexagons, regular octagons and regular pentagons. Under the drawings, describe which of the polygons tessellate. From your drawings, can you find a set of polygons in which all within the set tessellate? Show how you determined this.

G.2.5 Deduce formulas relating lengths and sides, perimeters, and areas of regular polygons and understand how limiting cases of such formulas lead to expressions for the circumference and the area of a circle.

Example: Use trigonometric functions to find the perimeter and the area of a regular 12-gon which has been inscribed in a circle of radius r .

G.2.6 Recognize and use coordinate geometry to verify properties of polygons such as regularity, congruence and similarity.

Example: Is the polygon formed by connecting the points $(2, 1)$, $(6, 2)$, $(5, 6)$, and $(1, 5)$ a square?

G.2.7 Develop simple geometric proofs involving congruent and similar polygons and provide reasons for each statement.

Example: Prove that the following pairs of polygons are similar.

[Insert a diagram with two sets of figures: (1) two regular hexagons, one with side lengths 7 and the other with side lengths 9 and (2) two quadrilaterals, each with one right angle and two obtuse angles, the first with sides 12, 8, 14 and 10, the second with sides 18, 12, 21, 15, both with appropriate equal angle marks, and each with a different orientation.]

Quadrilaterals

G.2.8 Describe, classify, and recognize relationships among the quadrilaterals such as squares, rectangles, rhombuses, parallelograms, trapezoids and kites.

Example: Use a drawing program to create a square, rectangle, rhombus, parallelogram, trapezoid, and kite. Judge which of the quadrilaterals has perpendicular diagonals and

draw those diagonals in the figures. Give a convincing argument that your judgment is correct

G.2.9 Prove and apply theorems about parallelograms and trapezoids (including isosceles trapezoids) involving their angles, sides, and diagonals and prove that given quadrilaterals are parallelograms, rhombuses, rectangles, squares, or trapezoids (as appropriate).

Example: Prove that both pairs of opposite sides of a parallelogram are congruent.

Triangles

G.2.10 Define, identify, construct, and solve problems involving perpendicular bisectors, angle bisectors, medians and altitudes in triangles.

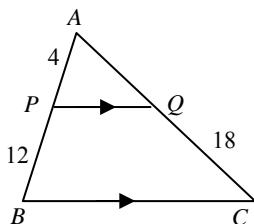
Example: Draw several triangles. Construct their angle bisectors. What do you notice?

G.2.11 Construct triangles congruent to given triangles, explaining and justifying the process used.

Example: Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.

G.2.12 Use theorems to show whether two triangles are congruent (SSS, SAS, ASA) or similar (AA, SAS, SSS).

Example: In the example below, prove that triangles ABC and APQ are similar and use the similar triangles to compute the length of the line segment AQ .



G.2.13 Apply the triangle inequality theorem.

Example: Can 7, 15, and 5 be the sides of a triangle? Explain how you know your answer is accurate.

G.2.14 Develop simple geometric proofs involving triangles and provide reasons for each statement.

Example: Prove that

- The sum of the angles in a triangle is 180° .
- The line joining the midpoint of two sides of a triangle is parallel to, and half the length of, the third side
- The perpendicular bisectors of the sides of a triangle meet at a point which is the center of the circle which contains the vertices of the triangle.

Isosceles Triangles

G.2.15 Prove and apply the isosceles triangle theorem and its converse.

Example: Given: $\triangle PQR$, PQ

Right Triangles

G.2.16 Prove the Pythagorean Theorem and its converse and use them to solve problems, including problems involving the length of a segment in the coordinate plane.

Example: Triangle DEF has vertices D(2,4), E(0,2), and F(3,1). Determine whether $\triangle DEF$ is a right triangle.

G.2.17 Prove and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle.

Example: In triangle ABC with right angle at C , draw the altitude \overline{CD} from C to \overline{AB} . Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem.

G.2.18 Use special right triangles ($30^\circ - 60^\circ$ and $45^\circ - 45^\circ$) to solve problems.

Example: An isosceles right triangle has one short side of 6 cm. Find the lengths of the other two sides.

G.2.19 Define and use the trigonometric functions (sine, cosine, tangent) in terms of angles of right triangles.

Example: In triangle ABC , $\tan A = \frac{1}{5}$. Find $\sin A$ and $\cos A$.

G.2.20 Deduce and apply the area formula $A = \frac{1}{2}ab \sin C$, where a and b are the lengths of two sides of a triangle and C is the measure of the included angle formed by the two sides.

Example: Find the area of an equilateral triangle with sides 5 units long.

G.2.21 Solve problems that can be modeled using right triangles, including problems that can be modeled using trigonometric functions. Interpret the solutions, and determine whether the solutions are reasonable, using technology as when appropriate.

Example: The force of gravity pulling an object down a hill is its weight multiplied by the sine of the angle of elevation of the hill. What is the force on a 3,000-pound car on a hill with a 1 in 5 grade? (A grade of 1 in 5 means that the hill rises one unit for every five horizontal units.)

Standard 3 **Circles**

G.3.1 Construct the circle that passes through three given points not on a line and construct tangents to circles and circumscribe and inscribe circles, justifying the processes used.

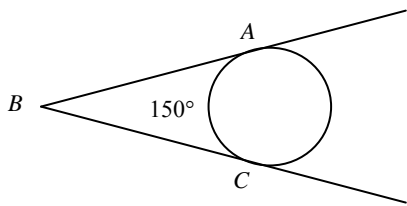
Example: Given a circle, find its center by drawing the perpendicular bisectors of two chords.

G.3.2 Define, deduce and use formulas for, and prove theorems for radius, diameter, chord, secant, and tangent.

Example: What is the angle between a tangent to a circle and the radius at the point where the tangent meets the circle?

G.3.3 Define, deduce and use formulas for, and prove theorems for measures of arcs and related angles (central, inscribed, and intersections of secants and tangents).

Example: Find the measure of angle ABC in the diagram below.



G.3.4 Define, deduce and use formulas for, and prove theorems for measures of circumference, arc length, and areas of circles and sectors.

Example: Use this theorem to find the sum of the interior angles of a convex n -gon.

G.3.5 Find the equation of a circle in the coordinate plane in terms of its center and radius and determine how the graph of a circle changes if a , b , and r are changed in the equation $(x - a)^2 + (y - b)^2 = r^2$.

Example: Find the equation of the circle with radius 10 and center $(6, -3)$.

G.3.6 Develop simple geometric proofs involving circles and provide reasons for each statement.

Example: Prove that an inscribed angle in a circle is half the measure of the central angle with the same arc.

Standard 4

Polyhedra and Other Solids

G.4.1 Identify, justify and apply properties of prisms, regular pyramids, cylinders, right circular cones and spheres.

Example: Which of these properties of a cylinder is not true?

- The bases are congruent;
- The sections produced by the intersection of a cylinder and two parallel planes are congruent;
- The volume is the product of the area of the base and the altitude
- The lateral area of a right circular cylinder is the product of the altitude and the base.

How do you know?

G.4.2 Solve problems involving congruent and similar solids.

Example: Explain how the surface area and volume of similar cylinders are related.

G.4.3 Find and use measures of sides, volumes, and surface areas of prisms, regular pyramids, cylinders, right circular cones and spheres. Relate these measures to each other using formulas.

Example: A marble is dropped into a glass that is roughly a right cylinder with a 6 cm diameter. The water level rises 1 mm. What is the volume of the marble?

G.4.4 Visualize solids and surfaces in three-dimensional space when given two-dimensional representations and create two-dimensional representations for the surfaces of three-dimensional objects.

Example: Make a net for a tetrahedron out of poster board and fold it to make the tetrahedron.

Standard 5

Geometric Reasoning and Proof

G.5.1 Describe the structure of and relationships within an axiomatic system (undefined terms, definitions, axioms/postulates, methods of reasoning, and theorems).

Example: Do you prove axioms from theorems or theorems from axioms?

G.5.2 Recognize that there are geometries, other than Euclidean geometry, in which the parallel postulate is not true and illustrate its counterparts in other geometries.

Example: Describe and illustrate at least one non-Euclidean geometry postulate.

G.5.3 Understand the difference between supporting evidence, counterexamples, and actual proofs.

Example: Draw and label a figure for the following conjecture. Support your conjecture with supporting evidence. Then write a simple proof for your conjecture.

If an angle bisector of a triangle is also an altitude, then the triangle is isosceles.

G.5.4 Develop simple geometric proofs (direct proofs, indirect proofs, proofs by contradiction and proofs involving coordinate geometry) using two-column, paragraphs, and flow charts formats and providing reasons for each statement in the proofs.

Example: Prove that the medians of a triangle meet at a point which is $\frac{2}{3}$ of the way from a vertex to the opposite side

Process Standards

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurement in problem situations.
- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication, and problem solving.
- The challenge is to ensure that technology supports-but is not a substitute for- the development of skills with basic operations, quantitative reasoning, and problem-solving skills.

- Graphing calculators should be used to enhance middle school and high school students' understanding and skills.
- The focus must be on learning mathematics, using technology as a tool rather than as an end in itself.

INTEGRATED MATHEMATICS I

Standard 1 **Algebra and Functions**

Relations and Functions

IM1.1.1 Determine whether a relation represented by a table, graph, words, or equation is a function or not a function.

Example: For a square of side x , the area y is given by $y = x^2$. Is y a function of x ? Is x a function of y ? Answer the same questions for $y = x^2$ if you are told that this holds for negative as well as positive values of x .

IM1.1.2 Identify the domain and range of relations represented by tables, graphs, words, and equations.

Example: What is the largest domain for x when $y = x^2$? What is the range of y in this case?

IM1.1.3 Solve problems that can be modeled using linear equations and inequalities, interpret the solutions, and determine whether the solutions are reasonable.

Example: A1 Taxi charges \$1 pickup plus 50 cents a mile and Ace Taxi charges 30 cents each half mile. For which distances does Ace Taxi cost less than A1 Taxi? Assume, for simplicity, that the charges per mile are proportional to the distance.

Linear Functions, Equations and Inequalities

IM1.1.4 Translate among various representations of linear functions including tables, graphs, words, and equations.

Example: Use a spreadsheet to create a table of values for the function $y = -\frac{1}{2}x + 5$. Graph the function.

IM1.1.5 Graph linear equations and show that they have constant rates of change.

Example: Kathy borrowed \$80 from her mother and plans to pay her mother \$10 per week until the debt is paid. The equation for the amount of money Kathy owes her mother is $y = 80 - 10x$, where x is the number of weeks after the loan. Graph the equation. What does the slope of the graph represent?

IM1.1.6 Determine the slope, x -intercept, and y -intercept of a line given its graph, its equation, or two points on the line.

Example: Find the slope and y -intercept of the line $4x + 6y = 12$.

IM1.1.7 Write, interpret, and translate among equivalent forms of equations for linear functions (slope-intercept, point-slope, and standard), recognizing that equivalent forms reveal more or less information about a given situation.

Example: Write the equation of the line $4x + 6y = 12$ in slope-intercept form. What is the slope of this line? Explain your answer.

IM1.1.8 Solve problems that can be modeled using linear equations and inequalities, interpret the solutions, and determine whether the solutions are reasonable.

Example: As your family is traveling along an interstate, you note the distance traveled every 5 minutes. The distance is approximately the same. You graph the distance traveled as a function of time, assuming that what was found for five minute time intervals holds for all time intervals up to two hours. Using the same value for each five minute interval, gives a linear graph. Predict the time of a journey of 50 miles. What does the slope of the graph represent?

IM1.1.9 Graph a linear inequality in two variables.

Example: Draw the graph of the inequality $6x + 8y \geq 24$ on a coordinate plane.

Pairs of Linear Equations and Inequalities

IM1.1.10 Understand the relationship between a solution of a pair of linear equations in two variables and the graphs of the corresponding lines and solve pairs of linear equations in two variables by graphing, substitution or elimination.

Example: Solve the system of equations: $2y + x = 10$ and $x = y + 3$. Graph the two lines, labeling the point of intersection.

IM1.1.11 Graph the solution set for a pair of linear inequalities in two variables with and without technology and use the graph to find the solution, including systems with no solution or infinitely many solutions.

Example Graph the inequalities $y \leq 4$ and $x + y \leq 5$. Shade the region where both inequalities are true

IM1.1.12 Solve problems that can be modeled using systems of linear equations containing two variables, interpret the solutions, and determine whether the solutions are reasonable.

Example: The income a company makes from a certain product can be represented by the equation $y = 10.5x$ and the expenses for that product can be represented by the equation $y = 5.25x + 10,500$, where x is the amount of the product sold and y is the number of dollars. How much of the product must be sold for the company to reach the break-even point?

Polynomials

IM1.1.13 Use the laws of exponents for variables with exponents. Multiply, divide, and find powers of variables with exponents.

Example: Simplify $a^2b^6(a^3)$, $(n + 2)(n - 2)$, and $(n + 2)^2$.

IM1.1.14 Add, subtract and multiply polynomials and divide polynomials by monomials.
Example: Subtract $(4x^2 - 7x + 2) - (x^2 + 4x - 5)$, multiply $(n + 2)(4n - 5)$, and divide $4x^3y^2 + 8xy^4 - 6x^2y^5$ by $2xy^2$.

IM1.1.15 Factor common terms from polynomials and factor quadratic expressions.
Example: Factor $4ax + 3ay + 4bx + 3$, $2x^2 - 7x + 3$, and $9a^2 - 4$.

Quadratic Equations and Functions

IM1.1.16 Graph quadratic functions.
Example: Draw the graph of $y = x^2 - 3x + 2$. Using a graphing calculator or a spreadsheet (generate a data set), display the graph to check your work.

IM1.1.17 Solve quadratic equations in the real number system with real number solutions by factoring, by completing the square, and by using the quadratic formula.
Example: Solve the equation $x^2 - x + 2 = 0$ in three ways, by factoring the polynomial, by the quadratic formula and by completing the square. Derive the general quadratic formula by applying the method of completing the square— to $ax^2 + bx + c = 0$.

IM1.1.18 Solve problems that can be modeled using quadratic equations, interpret the solutions, and determine whether the solutions are reasonable.
Example: A ball falls so that its distance above the ground can be modeled by the equation $s = 100 - 16t^2$, where s is the distance above the ground in feet and t is the time in seconds. According to this model, at what time does the ball hit the ground?

IM1.1.19 Analyze and describe the relationships among the solutions of a quadratic equation, the zeros of a quadratic function, the x -intercepts of a graph, and the factors of a quadratic expression.
Example: A graphing calculator can be used to solve $3x^2 - 5x - 1 = 0$ to the nearest tenth. Justify using the x -intercepts of $y = 3x^2 - 5x - 1$ as the solutions of the equation.

IM1.1.20 Sketch and interpret linear and non-linear graphs representing given situations and identify independent and dependent variables.
Example: The height (h) above water of a diver t seconds after she steps off a platform 100 feet high is given by the formula $h = 100 - 16t$. Graph the function.

Rational and Radical Expressions and Equations

IM1.1.21 Add, subtract, multiply, divide, reduce, and evaluate rational expressions with polynomial denominators. Simplify rational expressions with linear and quadratic denominators, including denominators with negative exponents.

Example: Simplify $\frac{x^2 - 4}{x^5} \div \frac{x^3 - 8}{x^8}$

IM1.1.22 Solve equations involving rational expressions.

Example: Solve $\frac{x+5}{4} = \frac{3x+5}{7}$ and $\frac{8}{x} + \frac{28}{x^2-4} = \frac{7}{x-2}$.

IM1.1.23 Simplify radical expressions involving square roots

IM1.1.24 Solve equations that contain radical expressions on only one side of the equation and identify extraneous roots when they occur.

Example: Solve the equation $\sqrt{x+6} = x$.

Standard 2 Geometry and Measurement

IM1.2.1 Find the length of line segments in one- or two-dimensional coordinate systems, the slopes of line segments in two-dimensional coordinate systems, and find the point that is a given fractional distance from one end of the segment to another.

Example: Find the length of the line segment joining A(3,8) and B(9,0). Find the midpoint of this segment, and the point that is 1/3 of the way from A to B.

IM1.2.2 Find and use measures of interior and exterior angles of polygons, justifying the method used.

Example: Calculate the measure of one interior angle of a regular octagon. Explain your method.

IM1.2.3 Solve problems involving congruent and similar polygons.

Example: Divide a regular hexagon into triangles by joining the center to each vertex. Show that these triangles are all the same size and shape and find the sizes of the interior angles of the hexagon.

IM1.2.4 Predict and describe the results of translations, reflections, and rotations on polygons. Describe a motion or series of motions that will show that two shapes are congruent.

Example: Use a drawing program to create quadrilaterals, regular hexagons, regular octagons, and regular pentagons. Under the drawings, describe which of the polygons tessellate. From your drawings, can you find a set of polygons in which all within the set tessellate? Show how you determined this

IM1.2.5 Deduce formulas relating lengths and sides, perimeters and areas of regular polygons and understand how limiting cases of such formulas lead to expressions for the circumference and area of a circle.

Example: Use the formula for the perimeter of a square to write a formula for the area of a square in terms of its perimeter.

IM1.2.6 Develop simple geometric proofs (direct proofs, indirect proofs, proofs by contradiction and proofs involving coordinate geometry) using two-column, paragraphs, and flow charts formats and providing reasons for each statement in the proofs.

Example: In triangle LMN , $LM = LN$. Prove that $\angle LMN \cong \angle LNM$.

Standard 3

Data Analysis and Probability

IM1.3.1 Organize and display data using appropriate methods to detect patterns and departures from patterns. Summarize the data using measures of center (mean, median) and spread (range, percentiles, variance, standard deviation). Compare data sets using graphs and summary statistics.

Example: Organize and display data using appropriate methods to detect patterns and departures from patterns. Summarize the data using measures of center and spread.
Example: Design and conduct a survey about the number of electronic games owned by girls and boys in your school. Organize and display the results of your survey in an appropriate graph. Describe the technique you used to get a random sample. Find the mean, median and mode of your survey data. Which of these gives a useful summary of the data?

IM1.3.2 Distinguish between random and non-random sampling methods, identify possible sources of bias in sampling, describe how such bias can be controlled and reduced, evaluate the characteristics of a good survey and well-designed experiment, design simple experiments or investigations to collect data to answer questions of interest, and make inferences from sample results.

Example: To determine what type of videos their customers liked, Drake Video surveyed every tenth person to walk in their store. Describe the sampling method used by Drake Video. Is it an unbiased sampling? Explain your answer

IM1.3.3 Evaluate reports based on data published in the media by considering the source of the data, the design of the study, and the way the data are analyzed and displayed.

Example: Find an example of a graph in a newspaper or magazine that could be considered misleading. Explain why the graph could be misleading.

Standard 4

Discrete Mathematics

IM1.4.1 Analyze and apply algorithms for searching (sequential, binary), for sorting (bubble sort, quick sort, bin sort) and for solving optimization problems.

Example: Use bubble sort to put 13, 14, 12, 11, 15 in increasing order.

IM1.4.2 Analyze and interpret relationships defined iteratively and recursively.

Example: Use the recursive definition of the Fibonacci numbers to find the fifth term.

IM1.4.3 Define arithmetic and geometric sequences recursively.

Example: There are 2,500 fish in a pond. Each year the population decreases by 25 percent, but 1000 fish are added to the pond at the end of the year. Find the population in 5 years. Also, find the long-term population.

IM1.4.4 Determine the number of ways events can occur using permutations, combinations, and the Fundamental Counting Principle.

Example: You are getting dressed one morning when you realize that you have far too many choices. You have 6 shirts to choose from, 4 pairs of jeans, and 3 pairs of shoes. Ignoring color coordination, construct a tree diagram or other pictorial representation to show how many different outfits you could assemble.

IM1.4.5 Determine whether two propositions are logically equivalent.

Example: Show that “If today is Sunday, then we have school tomorrow” and “It is not Sunday or we have school tomorrow” are logically equivalent.

IM1.4.6 Distinguish between inductive and deductive reasoning. Identify inductive reasoning as central to the scientific method and deductive reasoning as characteristic of mathematics.

Example: What type of reasoning are you using when you look for a pattern?

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INTEGRATED MATHEMATICS II

Standard 1

Algebra and Functions

IM2.1.1 Use and interpret function notation, including evaluation of functions represented by tables, graphs, words, equations or a set of ordered pairs.

Example: Given the function f below, find the indicated value, if possible.

- represented by the set of ordered pairs $\{(3,5), (2,-3), (1,7), (0,2)\}$, find the value of $f(1)$;
- where $f(x) = \sqrt{4-x}$, find the value of $f(2)$ and $f(8)$

IM2.1.2 Recognize and describe the relationships among the solutions of an equation, the zeros of a function, the x -intercepts of a graph, and the factors of a polynomial expression.

Example Solve the equation $x^4 + x^3 - 7x^2 - x + 6 = 0$, given that $x - 2$ and $x + 3$ are factors of $x^4 + x^3 - 7x^2 - x + 6$.

IM2.1.3 Analyze, describe, and sketch graphs of quadratic functions including the lines of symmetry.

Example: Find the zeros for $y = x^2 - 4$. If $y = x^2 - 4$ has a maximum or minimum value, give the ordered pair corresponding to the maximum or minimum point.

Standard 2

Geometry and Measurement

IM2.2.1 Find the lengths and midpoints of line segments in one- or two-dimensional coordinate systems and find the point that is a given fractional distance from one end of the segment to another.

Example: Find the length of the line segment joining A(3,8) and B(9,0). Find the midpoint of this segment, and the point which is $\frac{1}{3}$ of the way from A to B.

IM2.2.2 Construct congruent segments and angles, angle bisectors, perpendicular bisectors, and parallel and perpendicular lines using appropriate geometric construction tools, explaining and justifying the process used.

Example: Construct the perpendicular bisector of a given line segment, justifying each step of the process.

IM2.2.3 Recognize, use, and justify the relationships between special pairs of angles formed by parallel lines and transversals.

Example: In the diagram, the lines k and l are parallel. What is the measure of angle x ? Explain your answer.

IM2.2.4 Identify and apply properties of and theorems about parallel and perpendicular lines, and write equations of parallel and perpendicular lines, and develop simple geometric proofs involving parallel and perpendicular lines.

Example: Find an equation of a line perpendicular to $y = 4x - 2$ which contains the point (4,1).

IM 2.2.5 Identify, justify and apply properties of planes.

Example: Describe the intersection of plane R with parallel planes S and T .

IM.2.2.6. Deduce and apply the area formula $A = \frac{1}{2}ab \sin C$, where a and b are the lengths of two sides of a triangle and C is the measure of the included angle formed by the two sides.

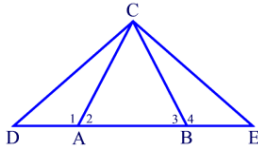
Example: Find the area of $\triangle DEF$ if $d = 3$, $e = 8$, and $C = 35^\circ$. Round the area to the nearest tenth.

IM2.2.7 Prove and apply the triangle inequality theorem.

Example: Can 7, 15 and 5 be the sides of a triangle? Prove your answer is accurate.

IM2.2.8 Prove and apply the isosceles triangle theorem and its converse.

Example: Given isosceles triangle ABC and $DA \cong EB$



Prove: $\triangle DAC \cong \triangle EBC$

IM2.2.9 Describe, classify, and recognize relationships among the quadrilaterals such as squares, rectangles, rhombuses, parallelograms, trapezoids, and kites.

Example: Use a drawing program to create a square, rectangle, rhombus, parallelogram, trapezoid, and kite. Judge which of the quadrilaterals has perpendicular diagonals and draw those diagonals in the figures. Give a convincing argument that your judgment is correct.

IM2.2.10 Develop simple geometric proofs involving congruent and similar polygons and provide reasons for each statement.

Example: Amanda shows that the similarity test for rectangles ($\frac{AB}{EF} = \frac{BC}{FG}$) does not work for all quadrilaterals.

IM2.2.11 Define, identify, construct and solve problems involving altitudes, medians, angle bisectors, and perpendicular bisectors in triangles.

Example: Draw several triangles. Construct their angle bisectors. What do you notice?

IM2.2.12 Prove the Pythagorean Theorem and its converse to solve problems, including problems involving the length of a segment in the coordinate plane.

Example: Triangle DEF has vertices $D(2,4)$, $E(0,2)$, and $F(3,1)$. Determine whether $\triangle DEF$ is a right triangle.

IM2.2.13 Use special right triangles ($30^\circ - 60^\circ$ and 45°) to solve problems.

Example: An isosceles right triangle has one short side of 6 cm. Find the lengths of the other two sides.

IM2.2.14 Define and use the trigonometric functions (sine, cosine, tangent) in terms of angles of right triangles.

Example: In triangle ABC , $\tan A = \frac{1}{5}$. Find $\sin A$ and $\cot A$.

IM2.2.15 Solve problems that can be modeled using right triangles, including problems that can be modeled using trigonometric functions. Interpret the solutions, and determine whether the solutions are reasonable using technology as appropriate.

Example: The force of gravity pulling an object down a hill is its weight multiplied by the sine of the angle of elevation of the hill. What is the force on a 3,000-pound car on a hill with a 1 in 5 grade? (A grade of 1 in 5 means that the hill rises one unit for every five horizontal units.)

IM2.2.16 Construct the circle that passes through three given points not on a line and construct tangents to circles and circumscribe and inscribe circles, justifying the process used.

Example: Given a circle, find its center by drawing the perpendicular bisectors of two chords.

IM2.2.17 Define, deduce and use formulas for, and prove theorems for radius, diameter, chord, secant, and tangent.

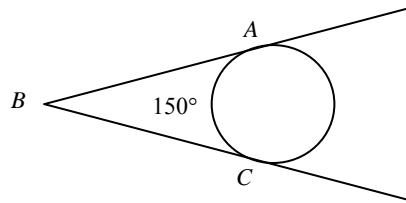
Example: What is the angle between a tangent to a circle and the radius at the point where the tangent meets the circle?

IM2.2.18 Develop simple geometric proofs involving circles and provide reasons for each statement.

Example: Prove that an inscribed angle in a circle is half the measure of the central angle with the same arc.

IM2.2.19 Define, deduce and use formulas for, and prove theorems for measures of arcs and related angles (central, inscribed, and intersections of secants and tangents).

Example: Find the measure of angle ABC in the diagram below.



IM2.2.20 Define, deduce and use formulas for, and prove theorems for measures of circumference, arc length, and areas of circles and sectors.

Example: Use an appropriate theorem to find the sum of the interior angles of a convex n -gon.

IM2.2.21 Identify, justify and apply properties of prisms, regular pyramids, cylinders, right circular cones and spheres.

Example: Which of these properties of a cylinder is not true?

- The bases are congruent.
- The sections produced by the intersection of a cylinder and two parallel planes are congruent.
- The volume is the product of the area of the base and the altitude.
- The lateral area of a right circular cylinder is the product of the altitude and the base.

How do you know?

Standard 3

Data Analysis and Probability

IM2.3.1 For bivariate measurement data, create a scatter plot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools.

Example: Measure the wrist and neck size of each person in your class and make a scatter plot. Interpret the correlation coefficient and the least squares regression line. Graph the residuals and evaluate the fit of the linear equation. Which line is a better fit? Explain your reasoning.

IM2.3.2 Display and analyze bivariate data where at least one variable is categorical.

Example: It is sometimes perceived that many retired people living in Florida live in mobile homes. Describe how you would gather, display, and analyze the data to determine if more people 65 and older lived in mobile homes in Florida.

IM2.3.3 Recognize how linear transformations of univariate data affect shape, center, and spread.

Example: Discuss whether you would use the mean or median to measure the center of each of the data below and why you made your particular choice.

- The yield of soybeans (bushels per acre) for a sample farm in Indiana.
- The prices of cars associated with each household in your neighborhood.

IM2.3.4 Calculate and interpret the correlation coefficient. Use the correlation coefficient and residuals to evaluate a “best-fit” line.

Example: Calculate and interpret the correlation coefficient for the linear regression model in the last example. Graph the residuals and evaluate the fit of the linear equation.

IM2.3.5 Construct sample spaces and probability distributions in simple cases and use them to solve problems.

Example: A couple plans to have children until they have a boy or until they have four children, whichever comes first. List the outcomes in the sample space for this experiment. What is the expected number of children for this couple?

IM2.3.6 Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risk of events.

Example: When a die is rolled 3 times, what is the probability of obtaining a 6, followed by an even number, followed by a 4?

IM2.3.7 Use permutations, combinations, and other counting methods to determine the number of ways that events can occur.

Example: There are 5 students who work in a bookshop. If the bookshop needs 3 people to operate, how many days straight could the bookstore operate without the same group of students working twice?

Standard 4

Discrete Mathematics

IM2.4.1 Use the properties of matrix addition, subtraction, and scalar multiplication to solve problems.

Example: The table below shows the number of bound books produced during one shift at two publishing companies. Write a matrix to represent one day's total output at the two plants. Use your results to find the difference between production totals at the plants. Which plant produces more bound books with no graphics?

	Hardback With graphic	Hardback No Graphic	Soft back With Graphics	Soft back no graphics
Publisher A	500	800	950	1900
Publisher B	600	700	1000	1600

IM2.4.2 Create matrices to organize and store data categorized by two variables and interpret the meaning of a particular entry in a matrix.

Example: Write a matrix C to represent the data in the table below. Find element c_{24} . What does this element represent?

The Types of Books Chosen by Mr. Smith's Language Arts Classes

	Fiction	Nonfiction	Science Fiction	Sports
Boys	15	20	18	22
Girls	18	17	10	15

IM2.4.3 Use the properties of matrix multiplication, including identity and inverse matrices, to solve problems.

Example: Explain how two matrices can be multiplied and what the dimensions of the product matrix will be.

IM2.4.4 Represent a system of equations in two or three variables as a matrix equation $\mathbf{Ax} = \mathbf{b}$ and use technology to find $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$.

Example: Alana's Boutique is selling faux pearls for the following prices:

2 grey faux pearls and 3 black faux pearls cost \$8.25

3 grey faux pearls and 4 black faux pearls cost \$11.25

Let x = the cost of one grey pearl.

Let y = the cost of one black pearl

Write the system as a matrix equation. Use technology to find the cost of one grey pearl and the cost of one black pearl.

IM2.4.5 Model and solve problems using matrices.

Example: To prepare for a dance, a school needs to rent 100 chairs, four large tables, and 10 punch bowls. Rental prices were collected from two rental shops with the following matrix representing the two rental shops:

	R_1	R_2
Chairs	\$2	\$2.50

Tables	\$20	\$15
Bowls	\$6	\$4

Which rental shop, R_1 or R_2 , has the lowest price for the group of items?

IM2.4.6 Use and interpret relational conjunctions (*and, or, not*), terms of causation (*if... then*) and equivalence (*if and only if*). Distinguish between the common uses of such terms in everyday language and their use in mathematics.

Example: Write “If today is Sunday, then we have school tomorrow” as a propositional statement by defining statements p and q and using symbols.

IM2.4.7 Use truth tables to determine the truth values of propositional statements.

Example: Find the truth table for $(p \rightarrow q) \rightarrow (q \rightarrow p)$.

IM2.4.8 Recognize syllogisms, tautologies, flawed reasoning, and circular reasoning.

Example: Is the following reasoning valid? How do you know?

Today is Sunday and we have school tomorrow. If today is not Sunday, then we have school tomorrow. Therefore, we do not have school tomorrow.

IM2.4.9 Construct and interpret directed and undirected graphs, decision trees, networks, and flow charts.

Example: There are two islands in the River Seine in Paris. The city wants to construct four bridges that connect each island to each side of the riverbank and one bridge that connects the two islands directly. The city planners want to know if it is possible to start at one point, cross all five bridges, and end up at the same point without crossing a bridge twice. Use a graph to help solve this problem.

IM2.4.10 Use critical-path analysis to solve scheduling problems.

Example: Write a critical task list for redecorating your room. Some tasks depend on the completion of others and some may be carried out at any time. Use a graph to find the least amount of time needed to complete your project

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Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication.

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
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- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurement in problem situations.
- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication, and problem solving.
- The challenge is to ensure that technology supports-but is not a substitute for- the development of skills with basic operations, quantitative reasoning, and problem-solving skills.
 - Graphing calculators should be used to enhance middle school and high school students' understanding and skills.
 - The focus must be on learning mathematics, using technology as a tool rather than as an end in itself.

INTEGRATED MATHEMATICS III

Standard 1 Algebra and Functions

Functions

IM3.1.1 Find the zeros, domain and range of a function.

Example: Draw the graph $y = x^3 - 3x^2 - x + 3$ then find the zeros of the function; i.e., find x when $y = 0$.

IM3.1.2 Use and interpret function notation, including evaluation of functions represented by tables, graphs, words, equations or a set of ordered pairs.

Example: Given the function, f , below, find the indicated value, if possible.

- represented by the set of ordered pairs $\{(3,5), (2,-3), (1,7), (0,2)\}$, find the value of $f(1)$;
- where $f(x) = \sqrt{4-x}$, find the value of $f(2)$ and $f(8)$

Linear and Absolute Value Equations, Inequalities and Functions

IM3.1.3 Solve systems of linear equations and inequalities in three variables by substitution and elimination.

Example: Solve the system of equations: $x - 2y + 3z = 5$, $x + 3z = 11$, $5y - 6z = 9$.

IM3.1.4 Solve problems that can be modeled using systems of linear equations in up to three variables, interpret the solutions, and determine whether the solutions are reasonable.

Example: Each week you can work no more than 20 hours all together at the local bookstore and the drugstore. You prefer the bookstore and want to work at least 10 more hours there than at the drugstore. Draw a graph to show the possible combinations of hours that you could work.

IM3.1.5 Graph piecewise-defined functions.

Example: Graph the function $f(x) = \begin{cases} x + 2 & \text{if } x \leq 0 \\ 3x - 1 & \text{if } x > 0 \end{cases}$

IM3.1.6 Solve equations and inequalities involving the absolute value of a linear function.

Example: Solve the inequality $|x - 5| \geq 8$ and graph the solution.

Quadratic Equations and Functions

IM3.1.7 Define, add, subtract, multiply and divide complex numbers. Represent complex numbers, and the addition, subtraction and absolute value of complex numbers, in the complex plane.

Example: Let $z = 7 - 4i$ and $w = 10 + 6i$. Graph z , w and $z + w$. Prove that the number 0, z , w and $z + w$ are the vertices of a parallelogram on the complex plane.

IM3.1.8 Solve quadratic equations in the complex number system.

Example: Solve $x^2 - 2x + 5 = 0$ over the complex numbers.

IM3.1.9 Analyze, describe, and sketch graphs of quadratic functions including the lines of symmetry.

Example: Find the zeros for $y = x^2 - 4$. If $y = x^2 - 4$ has a maximum or minimum value, give the ordered pair corresponding to the maximum or minimum point.

IM3.1.10 Determine how the graph of a parabola changes if a , b , and c changes in the equation $y = a(x - b)^2 + c$. Find an equation for a parabola given sufficient information.

Example: Write the equation of the parabola with vertex $(3, 6)$, y -intercept 2 in vertex form.

Polynomial Expressions, Equations and Functions

IM3.1.11 Analyze, describe, and sketch graphs of polynomial functions by examining intercepts, zeros, domain and range, and end behavior.

Example: Determine by inspection the end behavior of the graph of the function $f(x) = -2x^3 + x^2 + 4x - 5$.

IM3.1.12 Use the binomial theorem to expand binomial expressions raised to positive integer powers.

Example: Expand $(x + 2)^4$.

IM3.1.13 Perform arithmetic operations, including long division and division with remainders, on polynomials by others of equal or lower degree.

Example: Divide $2x^3 - 3x^2 + x - 6$ by $x^2 + 2$.

IM3.1.14 Factor polynomials completely and solve polynomial equations by factoring.

Example: Solve $x^3 + 27 = 0$ by factoring.

IM3.1.15 Use graphing technology to find approximate solutions for polynomial equations.

Example: Approximate the solution(s) of $x^4 - 3x^3 + 2x - 7 = 0$ to the nearest tenth.

IM3.1.16 Solve problems that can be represented or modeled using polynomial equations, interpret the solutions, and determine whether the solutions are reasonable.

Example: You want to make an open-top box with a volume of 500 cubic inches from a piece of cardboard that is 25 inches by 15 inches by cutting squares from the corners and folding up the sides. Find the possible dimensions of the box

Rational and Radical Expressions, Equations and Functions

IM3.1.17 Write a polynomial function of the lowest degree with real coefficients given its zeros.

Example: Write an equation that has solutions $x = 2$, $x = 5i$ and $x = -5i$.

IM3.1.18 Analyze, describe, and sketch graphs of rational functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.

Example: Find the equations of the horizontal and vertical asymptotes of the function

$$f(x) = \frac{x+1}{x+5}$$

IM3.1.19 Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions, including expressions with negative exponents in the denominator.

Example: Simplify $\frac{x^2-4}{x^5} \div \frac{x^3-8}{x^8}$.

IM3.1.20 Understand the properties of rational exponents and use the properties to simplify, multiply, divide, and find powers of expressions containing negative and fractional exponents. Relate expressions containing rational exponents to the corresponding radical expressions.

Example: Write the expression $\left(x^{\frac{1}{2}}y^{\frac{-2}{3}}\right)^6$ in simplest form. Assume all variables are positive.

IM3.1.21 Analyze, describe, and sketch graphs of square root and cube root functions by examining intercepts, zeros, domain and range, and end behavior.

Example: Graph the function $y = \sqrt{x} + 7$ and find the domain and range.

IM3.1.22 Solve equations that contain radical expressions and identify extraneous roots when they occur.

Example: Solve the equation $x = \sqrt{x} + 2$.

IM3.1.23 Solve problems that can be modeled using equations involving rational and radical functions, including problems of direct and inverse variation. Interpret the solutions, and determine whether the solutions are reasonable.

Example: Two students, working independently, can complete a particular job in 20 minutes and 30 minutes, respectively. How long will it take to complete the job if they work together at the same rate as when doing the job alone?

Exponential and Logarithmic Functions

IM3.1.24 Analyze, describe, and sketch graphs of exponential functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.

Example: Draw the graphs of the functions $y = 2^x$ and $y = 2^{-x}$.

IM3.1.25 Know that the inverse of an exponential function is a logarithm, use laws of exponents to derive laws of logarithms, and use the inverse relationship between exponential functions and logarithms and the laws of logarithms to solve problems.

Example: If you know that $\log(2) = a$ and $\log(3) = b$, find $\log(36)$ in terms of a and b .

IM3.1.26 Solve exponential and logarithmic equations.

Example: Solve the equation $\log_2 x = 5$.

IM3.1.27 Solve problems that can be modeled using exponential and logarithmic equations, interpret the solutions, and determine whether the solutions are reasonable using technology as appropriate.

Example: The population of a certain country can be modeled by the equation $P(t) = 50e^{0.02t}$, where P is the population in millions and t is the number of years after 1900. Find when the population is 100 million, 200 million, and 400 million. What do you notice about these time periods?

Sequences and Series

IM3.1.28 Write the recursive formula for arithmetic and geometric sequences and find specified terms of arithmetic and geometric sequences.

Example: Find the tenth term of the arithmetic sequence 3, 7, 11, 15 ...

IM3.1.29 Write the formula for the general term for arithmetic and geometric sequences and make connections to linear and exponential functions.

Example: Write the formula for the general term of the geometric sequence 2, 6, 18, 54, 162...

IM3.1.30 Find partial sums of arithmetic and geometric series.

Example: In the last example, find the sum of the first 10 terms.

IM3.1.31 Solve problems involving applications that can be modeled using sequences and finite arithmetic and geometric series. Interpret the solutions and determine whether the solutions are reasonable using spreadsheets as appropriate.

Example: A restaurant has square tables which seat four people. When two tables are placed together, six people can be seated. If 20 square tables are placed together to form one long table, how many people can be seated?

Standard 2

Geometry and Measurement

IM3.2.1 Identify and apply properties of and theorems about parallel and perpendicular lines and write equations of parallel and perpendicular lines.

Example: Find an equation of a line perpendicular to $y = 4x - 2$ which contains the point (4, 1).

IM3.2.2 Identify, justify and apply properties of planes.

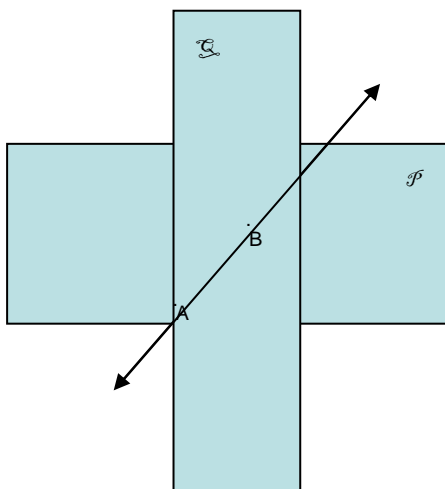
Example: Describe the intersection of plane R with parallel planes S and T .

IM3.2.3 Represent geometric objects and figures algebraically using coordinates, use algebra to solve geometric problems, and develop simple coordinate proofs involving geometric objects in the coordinate plane.

Example: Draw a triangle with vertices at $(1, 3)$, $(2, 5)$, and $(6, 1)$. Draw another triangle with vertices $(-3, -1)$, $(-2, 1)$, and $(2, -3)$. Are these triangles the same shape and size? Justify your answer.

IM3.2.4 Describe the intersection of two or more geometric figures in the plane.

Example: Write a description for the figure below



IM3.2.5 Construct triangles congruent to given triangles, explaining and justifying the process used.

Example: Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.

IM3.2.6 Prove and apply the triangle inequality theorem.

Example: Can 7, 15, and 5 be the sides of a triangle? Prove your answer is accurate.

IM3.2.7 Develop simple geometric proofs involving triangles and provide reasons for each statement.

Example: Prove that

- The sum of the angles in a triangle is 180° .
- The line joining the midpoint of two sides of a triangle is parallel to, and half the length of, the third side

IM3.2.8 Find the equation of a circle in the coordinate plane in terms of its center and radius and determine how the graph of a circle changes if a , b , and r are changed in the equation $(x - a)^2 + (y - b)^2 = r^2$.

Example: Find the equation of the circle with radius 10 and center $(6, -3)$.

IM3.2.9 Visualize solids and surfaces in three-dimensional space when given two-dimensional representations and create two-dimensional representations for the surfaces of three-dimensional objects.

Example: Make a net for a tetrahedron out of poster board and fold it to make the tetrahedron.

IM3.2.10 Find and use measures of sides, volumes, and surface areas of prisms, regular pyramids, cylinders, right circular cones and spheres. Relate these measures to each other using formulas.

Example: A marble cube is dropped into a glass that is roughly a right cylinder with a 6 cm diameter. The water level rises 1 mm. What is the volume of the marble?

Standard 3

Data Analysis and Statistics

IM3.3.1 Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions.

Example: About 30 percent of the students at a school are on the honor roll. If you took a random sample of 30 students, what range of students would likely to be on the honor roll?

IM3.3.2 Evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions. Interpret confidence levels and “margin of error.”

Example: In a random poll of 1025 women it was found that 47% of the women polled said they do not get enough time for themselves. The poll announced a margin of error of ± 3 percentage points for 95% confidence in its conclusion. Explain to someone who knows no statistics why it cannot be said that 47% of all adult women do not get enough time for themselves. Then explain what “95% confidence” means.

IM3.3.3 Compare the difference among surveys, experiments and observational studies and recognize which types of inferences can legitimately be drawn from each.

Example: Gather data to answer the question: Which area of the country has the highest high school dropout rate? Display your dropout data in various forms.

IM3.3.4 Compute basic statistics (mean, median, weighted mean, range, variance, standard deviation) and understand the distinction between a statistic and a parameter.

Example: Use spreadsheet formulas to compute measures that summarize your dropout data by state.

IM3 3.5 Understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term *variable*.

Example: Compare the data displayed in various forms in the first example. What do you notice about the impact the type of display has on the analysis of the data?

IM3.3.6 Use simulations to construct empirical probability distributions.

Example: Describe how you could use two coins to set up a simulation of a random phenomenon that has a 25% chance of a desired outcome.

IM3.3.7 Apply the properties of the normal distribution.

Example: Math SAT scores are normally distributed with mean 500 and standard deviation 100. What is the probability that Joan's SAT score is greater than 550?

IM3.3.8 Compute and interpret the expected value of random variables in simple cases.

Example: When you flip a coin 5 times, the number of heads is 0, 1, 2, 3, 4, or 5. Find the probability of each number of heads and draw a histogram of the results.

IM3.3.9 Compute the probability of compound events.

Example: An experiment consists of rolling a die 3 times and noting the number that lands on top at each throw. Find the probability of a 6, followed by an even number, followed by a 4.

IM3.3.10 Model and solve problems, including probability problems, using counting techniques.

Example: You know that your locker combination contains the numbers 2, 4, 6, and 8, but you have forgotten the order in which they occur. What is the probability that your locker opens with the first combination you try?

Standard 4

Discrete Mathematics

IM3.4.1 Know and use the concepts of sets, elements, and subsets.

Example: In the set of integers $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$, write the subset of even integers.

IM3.4.2 Perform operations on sets (union, intersection, complement, cross product).

Example: In the set of integers from 1 to 10, a subset is the prime numbers. Name the elements in this subset's complement.

IM3.4.3 Identify and give examples of undefined terms, axioms, and theorems.

Example: Do you prove axioms from theorems or theorems from axioms?

IM3.4.4 Describe logical statements using the terms *assumption*, *hypothesis*, *conclusion*, *converse*, *inverse*, and *contrapositive*. Find the converse, inverse, and contrapositive of statements.

Example: Find the converse of "If today is Sunday, then we have school tomorrow." Is the converse logically equivalent to the original statement? Explain.

IM3.4.5 Explain and illustrate the role of definitions, conjectures, theorems, proofs and counterexamples in mathematical reasoning. Construct logical arguments, assess the validity of logical arguments and give counterexamples to disprove statements.

Example: Find an example to show that triangles with two sides and one angle equal are not necessarily congruent.

IM3.4.6 Model and solve problems involving patterns using recursion and iteration, growth and decay, and compound interest.

Example: How many handshakes would occur in this room if everyone shook hands with everyone else? Create a spreadsheet that will find the number of handshakes starting with one person and increasing the number to 50.

IM3.4.7 Use mathematical induction to prove simple propositions.

Example: Use mathematical induction to prove the sum of the first n even positive integers is $n(n+1)$.

IM3.4.8 Use graph-coloring techniques to solve problems.

Example: Color a map of the Midwestern states of the United States so that no adjacent states are the same color. What is the minimum number of colors needed?

IM3.4.9 Use bin-packing techniques to solve problems of optimizing resource usage.

Example: Six large crates of electronic equipment are to be shipped to a warehouse. The crates weigh 2,800, 6,000, 5,400, 1,600, 6,800, and 5,000 pounds. Each delivery truck has a capacity of 10,000 pounds. What is the minimum number of trucks needed to send all the crates?

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PRE-CALCULUS

Standard 1

Relations and Functions

PC.1.1 Use paper and pencil methods and technology to graph polynomial, absolute value, rational, algebraic, exponential, logarithmic, trigonometric, inverse trigonometric and piecewise-defined functions, use these graphs to solve problems, and translate among verbal, tabular, graphical, and symbolic representations of functions using technology as appropriate.

Example: Draw the graphs of the functions $y = x^5 - 2x^3 - 5x^2$, $y = \frac{2x-1}{3x+2}$, $y = \sqrt{(x+2)(x-5)}$, and $f(x) = \sin^{-1}x$.

PC.1.2 Identify domain, range, intercepts, zeros, asymptotes, and points of discontinuity of functions represented symbolically or graphically, using technology as appropriate.

Example: Let $R(x) = \frac{1}{\sqrt{x-2}}$. Find the domain of $R(x)$ i.e., the values of x for which $R(x)$ is defined. Also, find the range, and asymptotes of $R(x)$

PC.1.3 Solve word problems that can be modeled using functions and equations.

Example: You are on the committee for planning the prom and need to decide what to charge for tickets. Last year you charged \$5.00 and 400 people bought tickets. Earlier experiences suggest that for every 20¢ decrease in price you will sell 20 extra tickets. Use a spreadsheet and write a function to show how the amount of money in ticket sales depends on the number of 20¢ decreases in price. Construct a graph that shows the price and gross receipts. What ticket price maximizes revenue?

PC.1.4 Recognize and describe continuity, end behavior, asymptotes, symmetry, and limits and connect these concepts to graphs of functions.

Example: Determine the numbers a and b so that the following function is continuous:

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ ax + b & \text{if } 1 < x < 2 \\ 5 - x & \text{if } x \geq 2. \end{cases}$$

PC.1.5 Find, interpret, and graph the sum, difference, product, and quotient (when it exists) of two functions, indicating the relevant domain and range of the resulting function.

Example: Find $(f + g)(x)$ if $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{x}{x-1}$. State the domain of $(f + g)(x)$.

PC.1.6 Find the composition of two functions, and determine the domain and the range of the composite function. Conversely, given a function, find two other functions the composition of which is the given one.

Example: If $h(x) = (2x + 3)^4$ find functions f and g so that $f \circ g = h$

PC.1.7 Define and find inverse functions, their domains and ranges, and verify whether two given functions are inverses of each other, symbolically and graphically.

Example: Find the inverse function of $h(x) = (x - 2)^3$.

PC.1.8 Apply transformations to functions and interpret the results of these transformations verbally, graphically, and numerically.

Example: Explain how you can obtain the graph of $g(x) = -|2(x + 3)^2 - 2|$ from the graph of $f(x) = x^2$.

Standard 2

Conics

PC.2.1 Derive equations for conic sections and use the equations that have been found.
Example: Derive an equation for the ellipse with foci at $(-1, 0)$ and $(1, 0)$ which contains the point $(0, 2)$?

PC.2.2 Graph conic sections with axes of symmetry parallel to the coordinate axes by hand, by completing the square, and find the foci, center, asymptotes, eccentricity, axes, and vertices (as appropriate).

Example: Graph $\frac{(x - 2)^2}{4} - \frac{(y + 3)^2}{9} = 1$. Find its foci, centers, asymptotes, eccentricity, axes and vertices (as appropriate).

Standard 3

Logarithmic and Exponential Functions

PC.3.1 Compare and contrast $y = e^x$ with other exponential functions, symbolically and graphically.

Example: Graph $y = e^x$, $y = 3^x$ and $y = 2^{-x}$. Show how to rewrite 3^x and 2^{-x} as e^{kx} for certain values of k .

PC.3.2 Define the logarithmic function $g(x) = \log_a x$ as the inverse of the exponential function $f(x) = a^x$. Apply the inverse relationship between exponential and logarithmic functions and the laws of logarithms to solve problems.

Example: Simplify the expression $e^{\ln 8}$

PC.3.3 Analyze, describe, and sketch graphs of logarithmic and exponential functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.

Example: For the function $l(x) = \log_{10}(x - 4)$ find its domain, range, x-intercept and asymptote, and sketch the graph.

PC.3.4 Solve problems that can be modeled using logarithmic and exponential functions. Interpret the solutions, and determine whether the solutions are reasonable.

Example: The amount A gm of a radioactive element after t years is given by the formula $A(t) = 100e^{-0.02t}$. Find t when the amount is 50 gm, 25 gm and 12.5 gm. What do you notice about these time periods?

Standard 4

Trigonometry

PC.4.1 Define and use the trigonometric ratios *cotangent*, *secant*, and *cosecant* in terms of angles of right triangles.

Example: Use the relationship between the lengths of the sides of a 30-60 right triangle to find the exact value of the secant of 30° .

PC.4.2 Model and solve problems involving triangles using trigonometric ratios.

Example: Find the area of triangle ABC if a , the side opposite angle A, measures 5 units, b , the side opposite angle B, measures 8 units and angle C measures 30 degrees.

PC.4.3 Develop and use the laws of sines and cosines to solve problems.

Example: You want to determine the location of a water tower by taking measurements from two positions 3 miles apart. From the first position, the angle between the water tower and the second position is 78° . From the second position, the angle between the water tower and the first position is 53° . How far is the water tower from each position?

PC.4.4 Define sine and cosine using the unit circle.

Example: Find the acute angle A for which $\sin 150^\circ = \sin A$.

PC.4.5 Develop and use radian measures of angles, measure angles in degrees and radians, and convert between degree and radian measures.

Example: Convert 90° , 45° , and 30° to radians.

Example: Find the length of an arc subtended by an angle of $\frac{5\pi}{6}$ radians on a circle of radius 5 cm.

PC.4.6 Deduce geometrically and use the value of the sine, cosine, and tangent functions at $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$, and $\frac{\pi}{2}$ radians and their multiples.

Example: Find the values of $\cos \frac{\pi}{2}$, $\tan \frac{3\pi}{4}$, $\csc \frac{2\pi}{3}$, $\sin^{-1} \frac{\sqrt{3}}{2}$, and $\sin 3\pi$.

PC.4.7 Make connections between right triangle ratios, trigonometric functions, and the coordinate function on the unit circle.

Example: Angle A is a 60° angle of a right triangle with a hypotenuse of length 14 and a shortest side of length 7. Find the exact sine, cosine, and tangent of angle A . Find the real numbers x , $0 < x < 2\pi$, with exactly the same sine, cosine, and tangent values.

PC.4.8 Analyze and graph trigonometric functions, including the translation of these trigonometric functions. Describe their characteristics (spread, amplitude, zeros, symmetry, phase, shift, vertical shift, frequency).

Example: Draw the graph of $y = 5 + \sin(x - \frac{\pi}{3})$.

PC.4.9 Define, analyze and graph inverse trigonometric functions and find the values of inverse trigonometric functions.

Example: Graph $f(x) = \sin^{-1}x$.

PC.4.10 Solve problems that can be modeled using trigonometric functions, interpret the solutions, and determine whether the solutions are reasonable.

Example: In Indiana, the length of a day in hours varies through the year, usually with the longest day of about 14 hours on June 21 and the shortest day of about 10 hours on December 21. Model this situation with a sine function, giving both the graph of this function and its formula. Find another day that is as long as July 4 according to your model.

PC.4.11 Derive the fundamental Pythagorean trigonometric identities, sum and difference identities, half-angle and double-angle identities and the secant, cosecant, and cotangent functions and use these identities to verify other identities and simplify trigonometric expressions.

Example: Find the acute angle between the lines given by $y = 2x$ and $y = 3x$.

PC.4.12 Solve trigonometric equations and interpret solutions graphically.

Example: Solve $3 \sin 2x = 1$ for x between 0 and 2π .

Standard 5

Polar Coordinates and Complex Numbers

PC.5.1 Define and use polar coordinates and relate polar coordinates to Cartesian coordinates.

Example: Convert the polar coordinate $(2, \frac{\pi}{3})$ to Cartesian coordinates.

PC.5.2 Represent equations given in Cartesian coordinates in terms of polar coordinates.

Example: Represent the equation $x^2 + y^2 = 4$ in terms of polar coordinates.

PC.5.3 Graph equations in the polar coordinate plane.

Example: Graph $y = 1 - \cos \theta$.

PC.5.4 Define complex numbers, convert complex numbers to polar form, and multiply complex numbers in polar form.

Example: Write $3 + 3i$ and $2 - 4i$ in trigonometric form and then multiply the results.

PC.5.5 Prove and use De Moivre's Theorem.

Example: Simplify $(1 - i)^{23}$.

Standard 6

Sequences and Series

PC.6.1 Define arithmetic and geometric sequences and series.

Examples: Write an example of each of the following sequences and series and explain their difference: arithmetic sequence, geometric sequence, arithmetic series and geometric series.

PC.6.2 Derive and use formulas for finding the general term for arithmetic and geometric sequences.

Example: Write the general term formula for the arithmetic sequence with initial term 2 and common difference 3.

PC 6.3 Develop, prove and use sum formulas for arithmetic series and for finite and infinite geometric series.

Example: Find the sum of the infinite geometric series $8 + 4 + 2 + \dots$

PC.6.4 Generate a sequence using recursion.

Example: Write the first five terms of the Fibonacci sequence with $a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. Observe a pattern of even and odd terms for this sequence. Prove that your observation is correct.

PC.6.5 Describe the concept of the limit of a sequence and a limit of a function. Decide whether simple sequences converge or diverge, and recognize an infinite series as the limit of a sequence of partial sums.

Example: Find the limit as $n \rightarrow \infty$ of the sequence $\frac{2n-1}{3n+2}$ and the limit as $x \rightarrow 5$ of the function $\frac{x^2-5^2}{x-5}$.

PC.6.6 Model and solve word problems involving applications of sequences and series, interpret the solutions and determine whether the solutions are reasonable.

Example: You put one grain of rice on the square of a chess board the first day, two on a second square the next day, and double the amount each day through the 64th day. How many grains of rice need to be used on the 64th square on the 64th day and how many grains of rice were used altogether?

PC.6.7 Derive the binomial theorem by combinatorics.

Example: Give an algebraic and combinatorial proof of the binomial theorem if a and b are real numbers and n is a positive integer.

Standard 7

Vectors and Parametric Equations

PC.7.1 Define vectors as objects having magnitude and direction and represent vectors geometrically.

Example: Graph the vector that goes from point $p = (3, 2)$ to the point $q = (5, 6)$.

PC.7.2 Use parametric equations to represent situations involving motion in the plane.

Example: Car A, traveling east at 40 mph, and Car B, traveling north at 30 mph, are heading toward the same intersection. Car A is 5 miles from the intersection when car B is 4 miles from the intersection. Write the parametric equations that describe the position of each car as a function of time.

PC.7.3 Convert between a pair of parametric equations and an equation in x and y

Example: Given the parametric equations $x = 3t^2$ and $y = t + 1$, find an equation relating x and y .

PC.7.4 Analyze planar curves, including those given in parametric form.

Example: Describe the curve that is defined by the parametric equation $x = 5 \cos t$, $y = 5 \sin t$ for $0 \leq t \leq \pi$

PC.7.5 Model and solve problems using parametric equations.

Example: For the problem in Standard 9.2 write a formula for the distance between the cars as a function of time and find when the cars are the closest.

Standard 8 Data Analysis

PC.8.1 Use linear models using the median fit and least squares regression methods. Decide which among several linear models gives a better fit. Interpret the slope in terms of the original context

Example: Measure the wrist and neck size of each person in your class and make a scatter plot. Find the median fit line and the least squares regression line. Which line is a better fit? Explain your reasoning.

PC.8.2 Calculate and interpret the correlation coefficient. Use the correlation coefficient and residuals to evaluate a “best-fit” line.

Example: Calculate and interpret the correlation coefficient for the linear regression model in the last example. Graph the residuals and evaluate the fit of the linear equation.

Process Standards

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Analyze and evaluate the mathematical thinking and strategies of others.

- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record, and communicate mathematical ideas.
- Select, apply, and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social, and mathematical phenomena.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurement in problem situations.
- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication, and problem solving.
- The challenge is to ensure that technology supports-but is not a substitute for- the development of skills with basic operations, quantitative reasoning, and problem-solving skills.
 - Graphing calculators should be used to enhance middle school and high school students' understanding and skills.
 - The focus must be on learning mathematics, using technology as a tool rather than as an end in itself.